MYP *by Concept*

1

Mathematics

Rita Bateson

Irina Amlin

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1

Mathematics

Rita Bateson
Irina Amlin
Series editor: Paul Morris

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From Rita Bateson: With thanks to my first and most enthusiastic maths teacher, my dad, Sean, and to my ever-supportive mother, Eileen, as well as to my wonderful siblings and friends who made me laugh (and feature) throughout. Special thanks to my proof-reading brother Sean and to one of the toughest mathematics teachers I ever had, Sarah Gibbons.

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From Irina Amlin: This book is dedicated to my two remarkable daughters, Penelope and Cora, who I know will each achieve great things in their own right. Thank you for giving up so much playtime with mummy to allow her to work on this!

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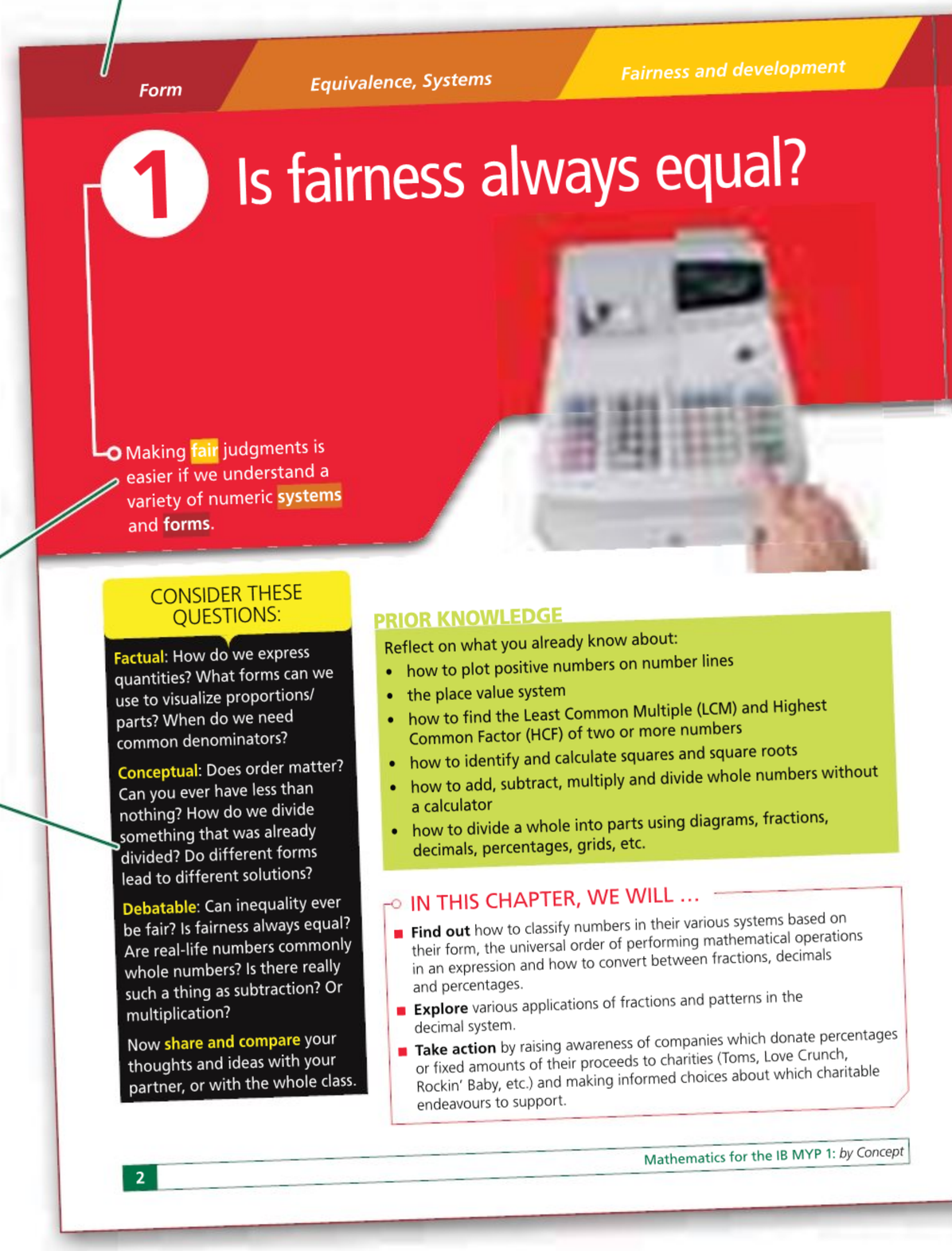
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How to use this book

Welcome to Hodder Education's *MYP by Concept* series! Each chapter is designed to lead you through an *inquiry* into the concepts of mathematics and how they interact in real-life global contexts.

Each chapter is framed with a *Key concept* and a *Related concept* and is set in a *Global context*.



The *Statement of Inquiry* provides the framework for this inquiry, and the *Inquiry questions* then lead us through the exploration as they are developed through each chapter.

KEY WORDS

Key words are included to give you access vocabulary for the topic. **Glossary** terms are highlighted and, where applicable, **search terms** are given to encourage independent learning and research skills.

As you explore, activities suggest ways to learn through *action.*

ATL

- Activities are designed to develop your *Approaches to Learning* (ATL) skills.

Assessment opportunities in this chapter:

- Some activities are *formative* as they allow you to practise certain of the MYP Mathematics *Assessment Objectives*. Other activities can be used by you or your teachers to assess your achievement against all parts of an assessment criteria.

Detailed information or explanation of certain points is given whenever necessary. Key Approaches to Learning skills for MYP Mathematics are highlighted whenever we encounter them.

Worked examples and practice questions are given in colour-coded boxes to show the level of difficulty:

Problem **Complex** **Challenging**

Each chapter covers one of the four branches of mathematics identified in the MYP Mathematics skills framework.



We have incorporated Visible Thinking – ideas, framework, protocol and thinking routines – from Project Zero at the Harvard Graduate School of Education into many of our activities. You are prompted to consider your conceptual understanding in a variety of activities throughout each chapter.

Finally, at the end of each chapter, you are asked to reflect back on what you have learned with our *Reflection table*, maybe to think of new questions brought to light by your learning.

Use this table to reflect on your own learning in this chapter.					
Questions we asked	Answers we found	Any further questions now?			
Factual					
Conceptual					
Debatable					
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?			
		Novice	Learner	Practitioner	Expert
Communication skills					
Critical-thinking skills					
Transfer skills					
Learner profile attribute(s)	Reflect on the importance of this attribute for your learning in this chapter.				
Knowledgeable					

Hint

In some of the Activities, we provide Hints to help you work on the assignment. This also introduces you to the new Hint feature in the on-screen assessment in MYP5.

! Take action

! While the book provides opportunities for action and plenty of content to enrich the conceptual relationships, you must be an active part of this process. Guidance is given to help you with your own research, including how to carry out research, how to make change in the world informed by Mathematics, and how to link and develop your study of Mathematics to the global issues in our twenty-first century world.

▼ Links to:

Like any other subject, mathematics is just one part of our bigger picture of the world. Links to other subjects are discussed.

● We will reflect on this learner profile attribute …

- Each chapter has an *IB learner profile* attribute as its theme, and you are encouraged to reflect on these too. We have explored the learner profile further with our feature, Meet a mathematician.

1 Is fairness always equal?

Making **fair** judgments is easier if we understand a variety of numeric **systems** and **forms**.



CONSIDER THESE QUESTIONS:

Factual: How do we express quantities? What forms can we use to visualize proportions/ parts? When do we need common denominators?

Conceptual: Does order matter? Can you ever have less than nothing? How do we divide something that was already divided? Do different forms lead to different solutions?

Debatable: Can inequality ever be fair? Is fairness always equal? Are real-life numbers commonly whole numbers? Is there really such a thing as subtraction? Or multiplication?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- how to plot positive numbers on number lines
- the place value system
- how to find the Least Common Multiple (LCM) and Highest Common Factor (HCF) of two or more numbers
- how to identify and calculate squares and square roots
- how to add, subtract, multiply and divide whole numbers without a calculator
- how to divide a whole into parts using diagrams, fractions, decimals, percentages, grids, etc.

IN THIS CHAPTER, WE WILL …

- **Find out** how to classify numbers in their various systems based on their form, the universal order of performing mathematical operations in an expression and how to convert between fractions, decimals and percentages.
- **Explore** various applications of fractions and patterns in the decimal system.
- **Take action** by raising awareness of companies which donate percentages or fixed amounts of their proceeds to charities (Toms, Love Crunch, Rockin' Baby, etc.) and making informed choices about which charitable endeavours to support.

World Cup Brazil 2014

Group A						Group B						Group C						Group D					
Country	W	D	L	GD	Pts	Country	W	D	L	GD	Pts	Country	W	D	L	GD	Pts	Country	W	D	L	GD	Pts
BRZ	2	1	0	5	**7**	NED	3	0	0	7	**9**	COL	3	0	0	7	**9**	CRC	2	1	0	3	**7**
MEX	2	1	0	3	**7**	CHI	2	0	1	2	**6**	GRE	1	1	1	–2	**4**	URU	2	0	1	0	**6**
CRO	1	0	2	0	**3**	SPA	1	0	2	–3	**3**	CIV	1	0	2	–1	**3**	ITA	1	0	2	–1	**3**
CMR	0	0	3	–8	**0**	AUS	0	0	3	–6	**0**	JPN	0	1	2	–4	**1**	ENG	0	1	2	–2	**1**

W – win **D** – draw **L** – lose **GD** – goal difference **Pts** – points

These Approaches to Learning (ATL) skills will be useful …

- Critical-thinking skills
- Creative-thinking skills
- Transfer skills

We will reflect on this learner profile attribute …

- **Caring** – We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.



Assessment opportunities in this chapter:

- **Criterion A**: Knowing and understanding
- **Criterion B**: Investigating patterns
- **Criterion C**: Communicating
- **Criterion D**: Applying mathematics in real-life contexts

THINK-PAIR-SHARE

Give examples of real-world situations in which you might need:

- **regular counting numbers from 1 onwards**
- **the number '0'**
- **a negative number**
- **a fraction**
- **a decimal.**

KEY WORDS

debt	per cent
difference	proportion
negative	sum

How do we express quantities?

THINK-PAIR-SHARE

There are three different number systems being used on pages 2 and 3. Can you find them?

EXTENSION

Can you **identify** any other number systems beyond those presented in this chapter? Search on the internet: **number systems beyond rationals**, **number systems around the world** and **number systems throughout history**.

NUMBERS



- Who taught you to count? Many children had the help of Sesame Street's Count von Count!



- A doughnut, or a symbol?



- A thermometer shows positive and negative integers



- Measuring cups use fractions and money uses decimals – all rational numbers!



- The number π (pi) is a non-recurring decimal: there is no pattern in the digits and there is no end!

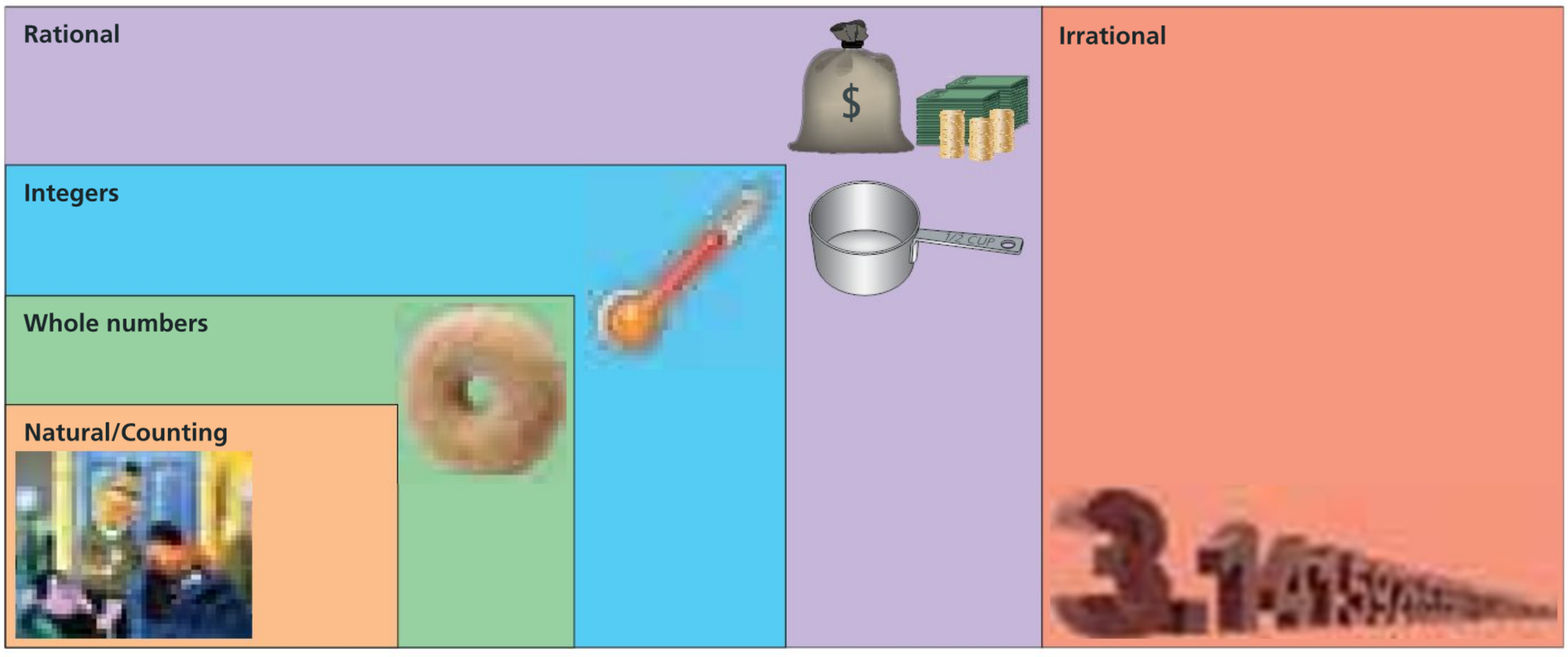
To say that numbers are used everywhere is not news. We don't, however, use all **types** of numbers all the time. It helps to **classify** numbers into various 'families' or **systems** and then choose the system that's most useful for the situation.

- **Natural numbers** – These are the numbers you learned as a child, possibly from children's programmes and apps. They are generally used for situations like **counting whole** objects.
- **Whole numbers** – As you got older, you realized that sometimes you have **nothing**, and you need a symbol to represent that too! Whole numbers are just natural numbers **with zero included in the list**.
- **Integers** – When can a number be less than zero? When you found that what you did during the day was affected by very cold temperatures, you were exposed to **negative numbers**. Integers include **all whole numbers, both positive and negative**.
- **Rational numbers** – Eventually, you began to learn about sharing. It is likely that this started by halving a treat with a friend or sibling. This would be your first use of a **fraction**. You may have also seen prices expressed with **decimals** to indicate values that were between whole numbers. **Rational numbers** include all **integers and everything in between**.
- **Irrational numbers** are any numbers which can't be expressed as a ratio (including as a fraction) and whose decimal never ends. Sometimes, they can be expressed as a square root (like root 2, $\sqrt{2}$), or even by a symbol (like π). But if you convert them to a decimal, you will always have a **never-ending string of non-recurring numbers**.

ACTIVITY: Number systems

ATL

- Communication skills: Use and interpret a range of discipline-specific terms and symbols



- Modelling how the number systems relate to each other

1 **Here is a set of numbers in a variety of different forms. Where could you place them on the diagram above to indicate which system they belong to? Could they belong to multiple sets?**

a) **–1**
b) **2.8**
c) **0**
d) $\sqrt{2}$
e) $\sqrt{49}$

2 **How could you use Russian babushka nesting dolls like those shown in the photograph to model the various number systems?**



Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

ACTIVITY: Timeline of number knowledge

ATL

- Transfer skills: Inquire in different contexts to gain a different perspective

Draw a timeline from birth to your current age and **label** the approximate ages at which you:

- **were able to recognize numbers when you saw them**
- **were able to count to 10**
- **were able to count to larger numbers**
- **understood what zero means**
- **first heard of negative numbers**
- **first understood what is meant by a fraction or a decimal.**

People's natural development of the understanding of numbers, your own included, takes the same course as the historical development of number systems. Just when you thought you knew your numbers, you realized you needed a new kind of number and learned about it!

Can you ever have less than nothing?

DISCUSS

Reflect briefly on your learning about number sets. Why do we need to be able to count below zero? **Discuss** what it means to be **less than zero** in each of the situations listed here:

- **in an elevator**
- **when measuring temperature**
- **at height or altitude**
- **counting money**
- **playing sports – think about goal difference or points difference**
- **time remaining (such as on iTunes).**

Hint

Which is the bigger number? People often see a pair of numbers such as –8 and 2, and refer to –8 as the bigger number. Though the ***digit*** 8 is bigger than the ***digit*** 2, the fact that the 8 is ***negative*** makes it much ***smaller*** than 2. Think about this ... who has more money – someone with $–8 or someone with $2?

ADDING WITH NEGATIVE NUMBERS

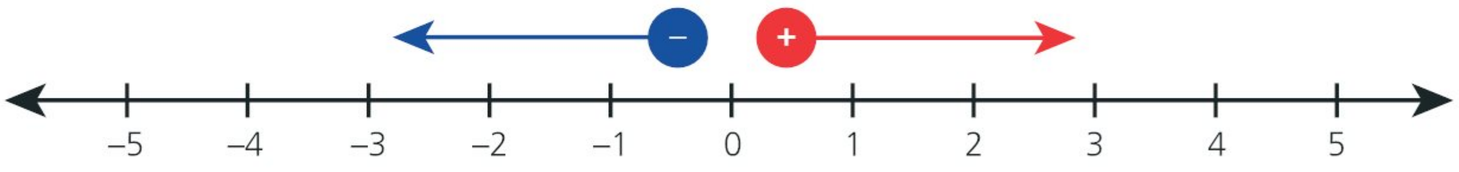
Suppose you owe a friend $5 for buying your lunch yesterday. You have –5 dollars. At home, you find you have $16 in your wallet. How much money will you have after you pay your friend back?

–5 (yesterday's loan) + (+16) (money in wallet)

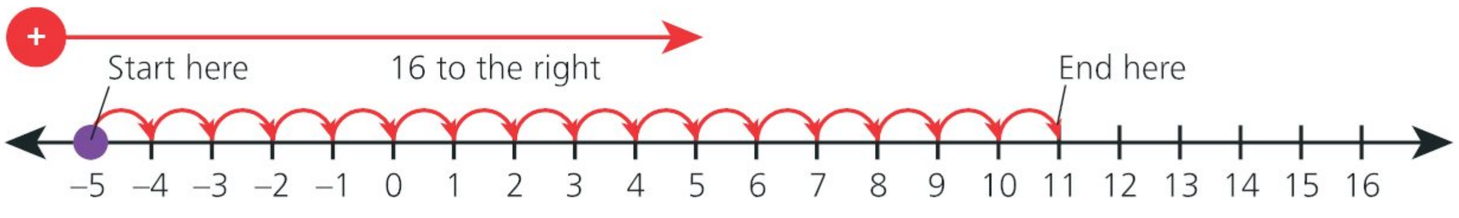
There are a few ways to approach a question like –5 + (+16), which we will look at next.

1. Using a number line

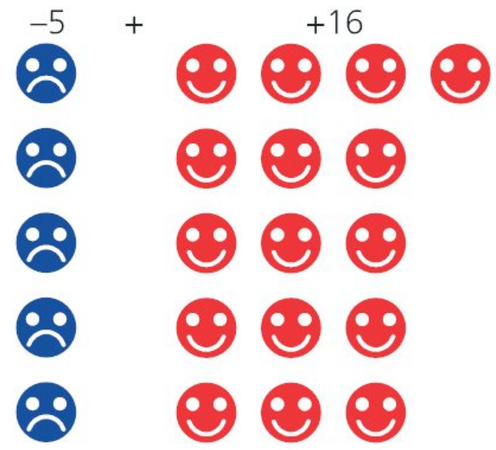
When adding integers using a number line, all we need to remember is that, if we are adding a negative, we move left, and when adding a positive we move right.



The diagram below shows how to **apply** the number-line method to our problem: adding –5 and +16.

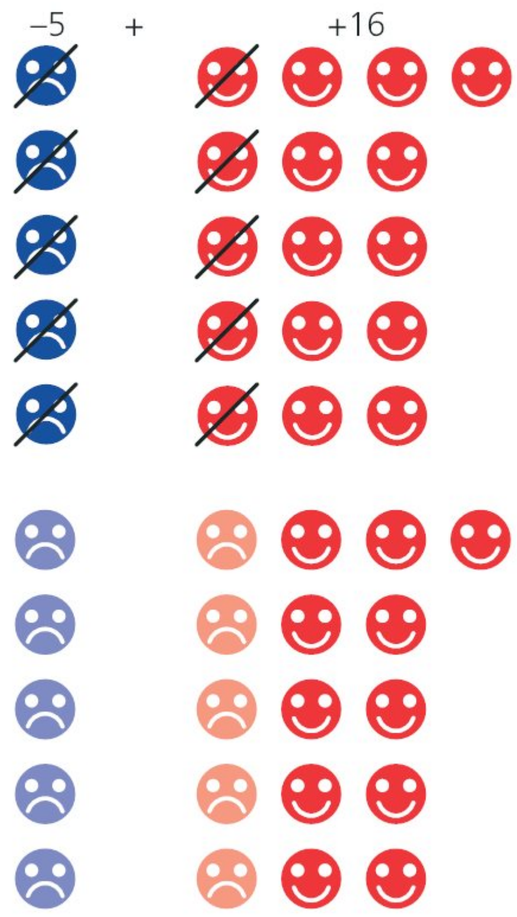


2. Battle of the blobs



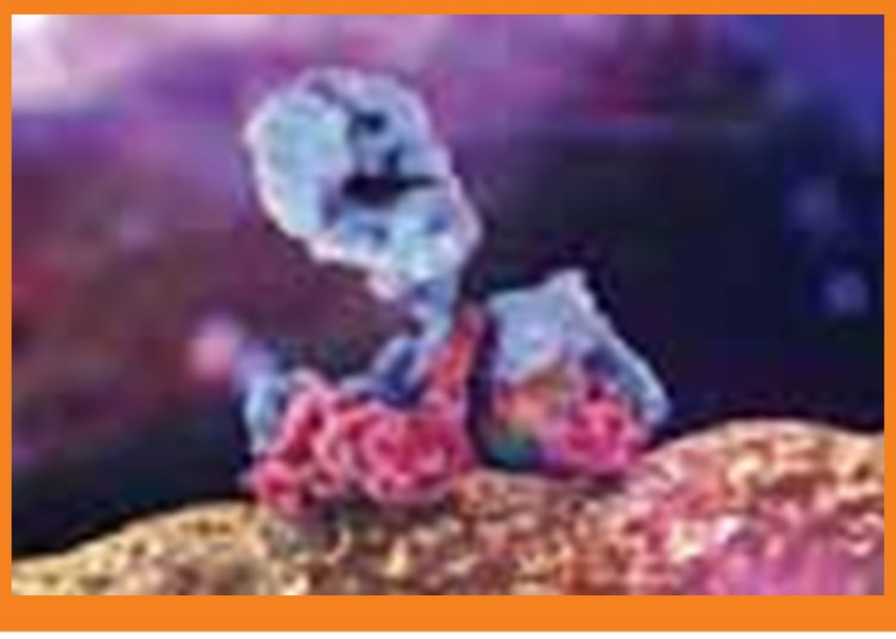
Each angry blue blob represents a negative and each happy red blob is a positive. When two blobs of opposite sides or colours get together in battle, they destroy each other – or 'cancel out'. They will both disappear. After a battle of blobs, only one side is left standing (this can be either the positive or negative side). We count up the remaining blobs, and there is our answer!

The diagram below shows how to **apply** this method to our problem.



Links to: Sciences

Our bodies produce Y-shaped antibodies that bind to bacteria or viruses – 'cancelling them out' to keep us healthy.



Hint

Blobs taking too long to draw? Try using squares and circles, noughts and crosses, or blue dots and red dots. You could devise your own quicker drawing system for 'battle of the blobs'.

3. Adding integers song

When in doubt and you need an easy way out … there is a song which tells you nothing about the reasoning but helps you answer the question! Search online for **'Row, row, row your boat' mathematics song**.

Same signs add and keep

Different signs subtract

Take the sign of the bigger number

Then you'll be exact.

If adding two positives or two negatives, the sign will stay the same, and you need only add the digits together.

If adding quantities with opposite signs, subtract the smaller quantity from the larger one and …

Your answer will be negative if the bigger quantity is negative, and positive if the bigger quantity is positive.

How will this work with our problem?

Same signs add and keep

Different signs subtract

Take the sign of the bigger number

There is a –5 and a +16. The signs are not the same, so we continue …

OK, so we subtract 16 – 5 = 11

16 is bigger than 5, and 16 is positive, so our answer will be positive.

The answer is +11.

ACTIVITY: Adding with negative numbers

ATL

- Communication skills: Reading, writing and using language to communicate information
- Critical-thinking skills: Consider ideas from multiple perspectives, use models and simulations to explore complex systems

What if, instead of finding money in your wallet, you owed your friend $5 for lunch and then had to **borrow** $16 for a taxi ride home?

Work out how much you owe using the following methods:

- **number line**
- **battle of the blobs**
- **song.**

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding, and Criterion C: Communicating.

ADDING LARGER NEGATIVE NUMBERS

Number lines are a great way to understand the relationship between positive and negative numbers, but they are limiting when you get into larger numbers. For example, consider this sum:

–350 + 210

You would need a long number line to solve it! But we can still use either of the other two methods.

Battle of the blobs

Although it would be time-consuming to draw all of these blobs, we know that 210 of them will cancel out. How can we figure out how many would be left, without drawing every blob? We can use a table.

Positives	Negatives
210	350
Cancel out 210	Cancel out 210
Left with 0	Left with 350 – 210
	= 140

So our final answer would be –140.

Adding integers song

Using the lyrics of the integers song:

- The signs are different, so we will subtract.
- 350 is a larger number than 210, so we can use the sign of '350', which is negative.
- 350 – 210 = 140
- Final answer: –140.

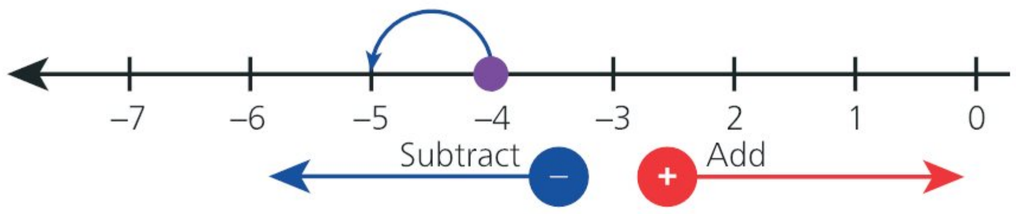
SUBTRACTING POSITIVE NUMBERS

WHAT MAKES YOU SAY THAT?

Can all three methods work in just the same way when subtracting a positive number? Use –4 – (+1) to **justify** your response.

Hint

Subtracting a positive number means we have lost a happy red blob before battle – it converted to the blues!



Look at the diagram above. Did you notice that the sum –4 – 1 would **yield** an equal result? This is because 1 and +1 are actually the same number, so subtracting 1 is the same as subtracting +1. We rarely include the '+' when we write positive numbers – it is simply assumed.

SUBTRACTING NEGATIVE NUMBERS

If you owe someone $20, you have –$20. Suppose your grandmother takes pity on you and gives you the $20. This can be written as:

–$20 (what you owe) + $20 (what your grandma gives you) = $0 (what you actually have)

Another way to look at this is that your grandmother **took away your debt**. In other words:



When you take away an amount equal to your debt, you are left with zero.

So if –$20 + $20 = $0 and –$20 – (–$20) = $0, we can conclude that **subtracting a negative number is the same as adding the positive version of that number**.

Now consider that 13 – (–27) is the same as 13 + 27, which is 40. The diagram shows how our blob friends handle this.



Since we are starting with 13 red blobs and no blue blobs, the reds have nobody to fight and there is nothing to cancel out. Since there are no blue blobs to be taken away (subtracted), we gain a new red for each negative we would have otherwise destroyed and cancelled. We gain 27 positives and we are left with 40 red.

To bracket or not to bracket?

When first introduced to integers, it is common to be puzzled about when to use brackets with positive and negative signs. The general rule is this: we can't write two **operations** (+, –, ×, ÷) next to one another as this is confusing. An expression such as 3 – (–10) has two '–' symbols, but only the first is an operation. The second is a negative sign. To minimize confusion, we can combine the two symbols to give a single operator, so there is no need for brackets.

EXTENSION

A mathematician might say that there is no such thing as subtraction. How can you use what you know about negative numbers to **justify** this statement?

Summary

When a question has an operation (such as +, –, × or ÷) next to a positive or negative sign (usually with brackets), you should combine these into one operation. Use these rules:

- If the operation and the sign are the same, they combine to give a '+'.
- If the operation and the sign are different, they combine to give a '–'.

4 + (+5) = 4 + 5 (we don't normally write a '+' in front of a positive number, it is **implied** (assumed))

4 – (–5) = 4 + 5

4 + (–5) = 4 – 5 (think of this as adding debt to a positive amount – adding debt means taking away money)

4 – (–5) = 4 + 5 (think of this as receiving a refund – you already had $4 and received a refund of $5, bringing you to $9)

In mathematics, a number in front of a set of brackets indicates multiplication. For example:

- 'three groups of water bottles' can be written as 3(water bottles).
- 'three times five' can be written as 3(5) or (3)(5).
- 'three times (1 + 4)' can be written as 3(1 + 4) or (3)(1 + 4).

PRACTICE EXERCISE

Calculate:

1 -4×4 (the signs are opposite)

2 $-5 \times (-10)$ (the signs are the same)

3 $12 \times (-1)$

4 $9 \times (-7)$

5 $(-2)(3)(-8)$

6 $(-6)^2$

MULTIPLICATION WITH INTEGERS

To multiply integers, first **determine** the **product** of the digits and then **determine** whether the sign will be positive or negative.

ACTIVITY: Visual multiplication of integers or 'extending your blob understanding'

ATL

- Transfer skills: Using skills and knowledge in multiple contexts

Use the blobs to **show** why 4×3 is 12.

Hint

Remember that this calculation can be **interpreted** as '3 groups of 4' or '4 groups of 3'.

Now use a diagram to depict -4×3. How many blobs do you have, and what colour are they?

Use your calculator to solve:

$6 \times (-3)$ $\quad -6 \times (-3)$ $\quad 6 \times 3$ $\quad 6 \times (-3)$

Can you come up with a general rule about multiplying two numbers with the same sign, or two numbers with different signs? **Justify** your response.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns.

DIVISION WITH INTEGERS

The same rules apply to division with integers as for multiplication. First, ignore the signs and **determine** the quotient, then **determine** the sign of your final answer, remembering that same signs give a positive and different signs give a negative result. But **never** use this rule for addition and subtraction!

Let's look at an example:

$(-56) \div 7$

Note that the numbers in this calculation have opposite signs.

$(-56) \div 7 = -8$

$(7 \times 8 = 56; \quad 7 \times (-8) = -56)$

Summary

Addition/subtraction	**Number line** Plot first number, move right to add and left to subtract.	**Battle of the blobs** Cancel out one-for-one. Who is left standing?	**Song** Same signs add and keep …
Multiplication/division	**Perform operation, then choose sign** Opposite signs give a negative answer; same signs give a positive answer		

USING SYMBOLS TO REPRESENT ACTIONS

You will already be familiar with adding, subtracting, multiplying and dividing, as well as with many other mathematical actions. These actions are called **operations**, as we saw on page 9, and can be represented by a symbol such as + or ×.

An interesting mathematical symbol, commonly shown as an exclamation mark '!', is called a **factorial**. The table shows you what to do if you see this symbol placed after a number.

Factorial	What this means in symbols	What this means in words	=
3!	3 × 2 × 1	Multiply all the natural numbers from 3 down to 1	= 6
4!	4 × 3 × 2 × 1	Multiply all the natural numbers from 4 down to 1	= 24
7!	7 × 6 × 5 × 4 × 3 × 2 × 1	Multiply all the natural numbers from 7 down to 1	= 5040

Another mathematical symbol indicates **absolute value** and is represented by two parallel lines $||$. When you see a number between these lines, it indicates that only the positive value is used, regardless of whether there is a plus or minus sign. This is sometimes referred to as magnitude. For example:

- $|-10|$ gives an answer of 10
- $|10|$ **also** gives an answer of 10.

ACTIVITY: Time to operate

ATL

- Creative-thinking skills: Apply existing knowledge to generate new ideas

1 **A class was practising operations using symbols and wanted to create and use their own system.**

Samir decided to create his own operation symbol using '¤'.

He created some examples to communicate what this symbol would mean:

- **3 ¤ 5 = 4**
- **2 ¤ 10 = 6**
- **0 ¤ 100 = 50**

Can you determine what the symbol means? Can you explain it in words? Can you give other examples using this symbol?

2 **Vanessa created a particular meaning for the symbol §. When she placed it before a number, she performed an action and came up with a final number. Here are some examples:**

- **§456272 = 42**
- **§1111192929291 = 11**
- **§9888888888888888888882 = 92**

Can you discover what the symbol § does to the number? Can you explain it in words? Can you give other examples using this symbol?

3 **Na'ilah invented a new use for the symbol #. She decided the symbol would mean you must multiply the two numbers used with the symbol and add 1 to the total. Formulate some examples using this symbol in this way.**

4 **Now it's your turn. Create a new 'operation' which can be done on a pair of numbers. Choose a symbol which will represent the action in your mind. You can use a symbol which already exists for this new purpose or you can create your own new symbol. Show your new symbol and explain how it operates. Give examples – both simple and challenging numbers should be used to explore this new idea.**

Assessment opportunities

- This activity can be used to assess Criterion B: Investigating patterns and Criterion C: Communicating.

Does order matter?

WHAT MAKES YOU SAY THAT?

Pick any natural number under 20. Double it, then add 50.

Take the same initial number, add 50, then double it.

What do you notice? What was different about the two tasks?

Links to: Language and Literature

Think of the order of mathematical operations as being like sentence structure or even punctuation. A misplaced comma can change the entire meaning of a sentence. Likewise, misplaced brackets can change the answer to a problem.



Adding, subtracting, multiplying, and dividing are examples of operations – procedures for evaluating a mathematical expression or problem.

When we have an expression with more than one operation, which one must be done first? How can we decide the order of operations and will it affect the answer? It is important that problems are solved in the correct order, so our answers are correct and match those of another person solving the same problem.

Mathematicians have developed accepted **conventions**, or rules, for the order in which we complete operations. These conventions have been built into computer programming languages so that programs, like spreadsheets and calculators, follow the same logical steps. Scientists also apply these rules when measuring and calculating relationships.

Mathematicians agree that operations should be performed in the following order:

1. **Exponents**/**Powers**/**Indices** and **roots**
2. **Multiplication** and **division**
3. **Addition** and **subtraction**

(**But** if something appears like this, in a bracket or parentheses, you must do that operation first of all, before any other operation.)

We can **use** an acronym like BEDMAS to help us remember the order of operations we must always follow. Each letter refers to one operation: **B**rackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction.

Examples

1 $4 + 1 \times 5$ **2** $3 \times (-6 - 2)$

Solutions

1 Looking at the checklist above, there are no exponents, so we will multiply first:

$1 \times 5 = 5$

Then, we add 4 and 5:

$4 + 5 = 9$

Performing the operations in the order shown would have given this answer:

$4 + 1 \times 5 = 5 \times 5$

$= 25$

A **very** different answer!

2 $3 \times (-6 - 2) = 3 \times (-8)$

$= -24$

Why was subtraction performed before multiplication this time?

ACTIVITY: Mathematical operations

ATL

- Transfer skills: Inquire in different contexts to gain a different perspective

Six of one, half dozen of the other

Research some different terms for mathematical operations. **Reflect** on the international-mindedness of mathematics as a language.

Although mathematics is often said to be a universal language, many different words can be used to express similar ideas. For example, these () can be called brackets or parentheses and in x^2 the '2' can be called an exponent, a power, an index or an order. The name that is used often depends where and when the symbol is being used.

- **Can you find different symbols that represent the same operation, or is there only one?**
- **Can you find different names for the same operations, or is there only one for each?**

Consider whether different forms lead to different solutions. Although the order of operations is the same around the world, the acronyms and mnemonic devices people use to remember the sequence can be different. For instance, Bring Everyone Dinner, Many Are Starving for BEDMAS.

Choose a mnemonic device you will use, or create one of your own.

Imagine people from all over the world use their preferred mnemonic device to **solve** this problem:

$$(2 + 4)\ 2 \times (10 - 5) \div 4$$

Assuming they make no mistakes, will they all come up with the same answer? **Justify** your answer by giving examples and explanations for your decision.

Are there any operations which can change the order of the other operations within the problem? Give an example problem to **show** how operations can change order.

Remember to:

- **Communicate using mathematical language**
- **Communicate your lines of reasoning (don't leave anything out that shows your thinking).**

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

EXTENSION

Type $(-5)^6$ in your calculator. Now type -5^6. Why aren't these answers equal?

What forms can we use to visualize proportions?

A **proportion** is a part of a whole. It is often expressed as a fraction (half a cup), a percentage (10% off the full price) or a decimal (0.03 francs). Sometimes it is difficult to grasp what kind of amount you are dealing with, unless you visualize it. Let's explore some different ways of visualizing fractions, decimals and percentages.

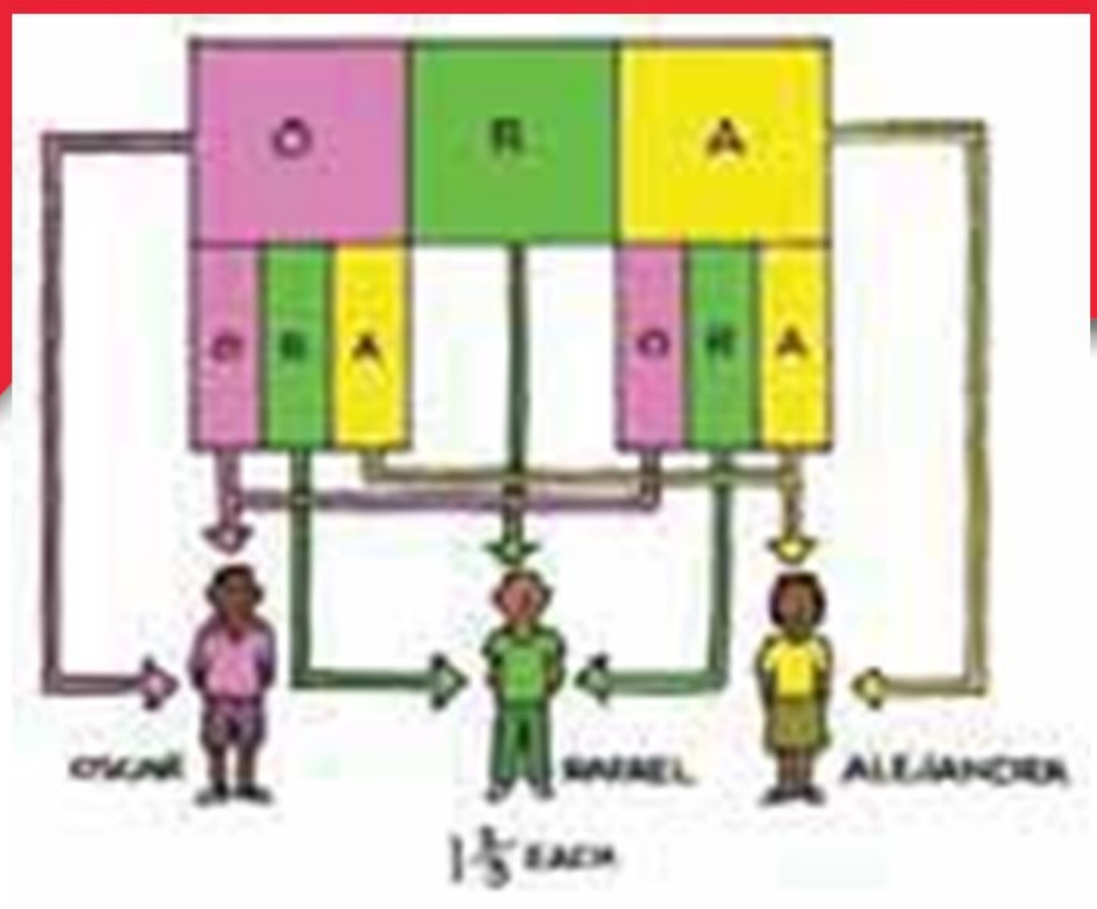
Look at the diagrams. How can Oscar, Rafael and Alejandra divide their work equally if they are planting trees over five equal areas of land that need reforestation? How can we use drawings to represent the amount of land each person works?



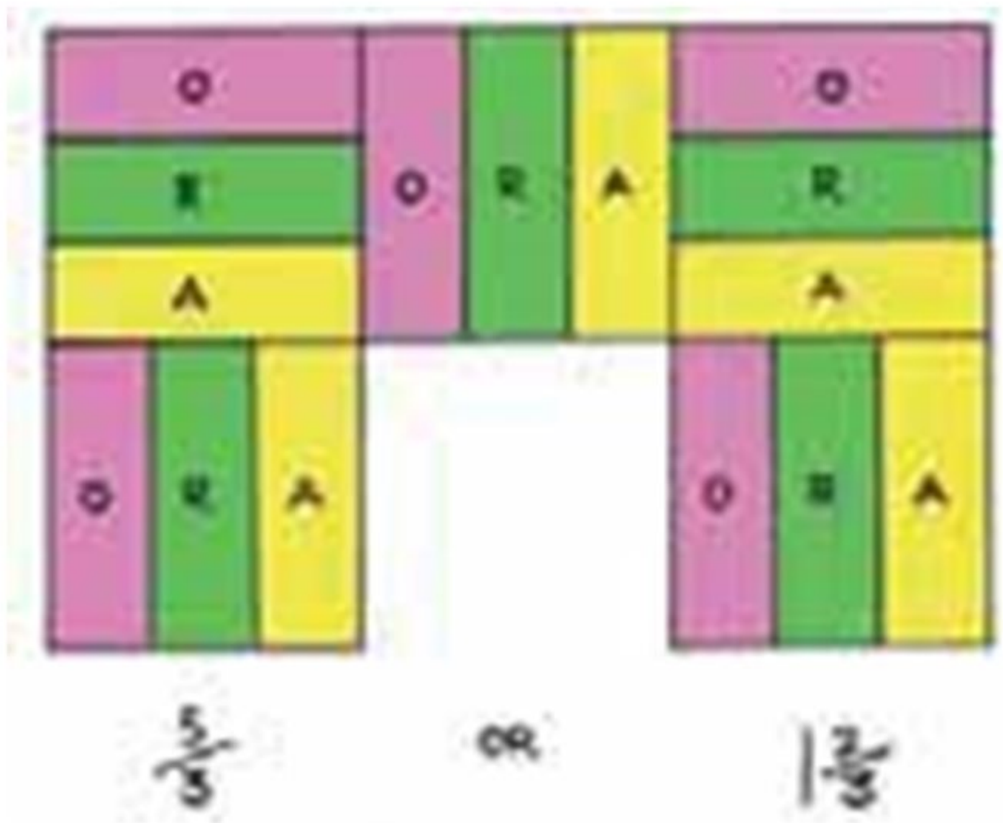
Draw the five equal areas:



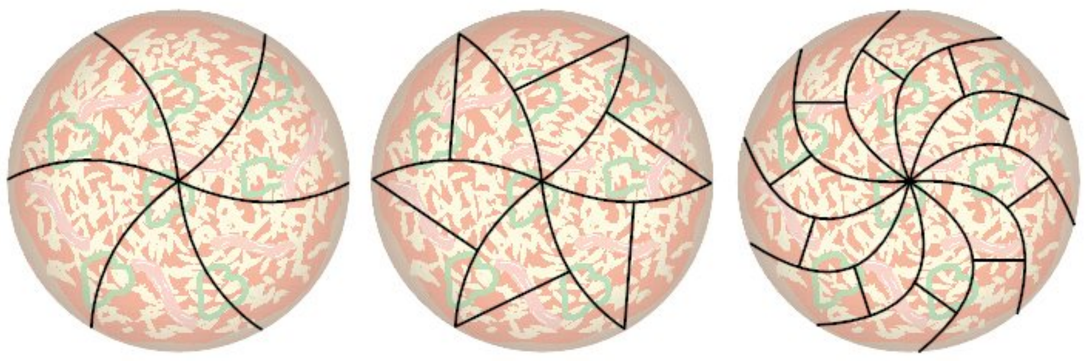
One way to divide the work is to let each person take one whole plot, until there are fewer than three left. Then they share the remaining land between them, one area at a time.



Here, each person takes a third of each plot of land, and we combine their respective thirds.



Both ways of representing these fractions give equal answers!



- Pizzas sliced in non-traditional ways – mathematicians invent new ways to slice pizza

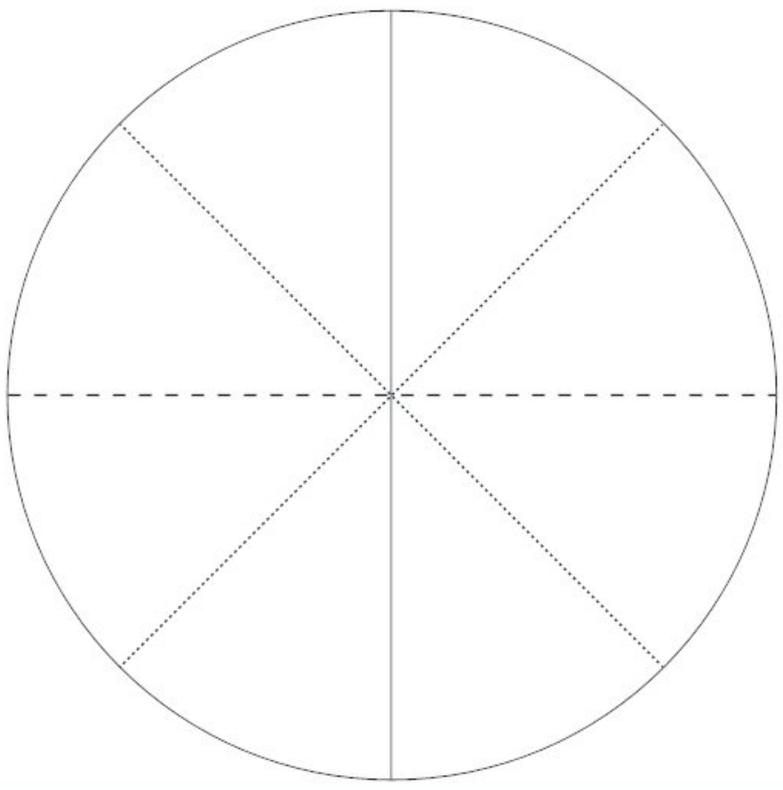
Although the methods of dividing the pizzas pictured above into fractions may look cool and complex, how do they **compare** to straightforward or simplified fractions? Do they help us to make division more fair?

THINK-PUZZLE-EXPLORE



1 **Copy** the diagram above and shade in $\frac{1}{4}$ of it.
2 **Draw** it again, but this time shade in $\frac{2}{8}$.
3 **Draw** it one last time, and shade in $\frac{4}{16}$.
What do you notice?
4 **Try the same exercise with the following shapes and pairs of fractions.**

a $\frac{1}{2}$ and $\frac{4}{8}$



b $\frac{2}{3}$ and $\frac{4}{6}$.



Now ignore the images you produced and think about the numbers you have been working with. How are the numerators related? How are the denominators related?

- $\frac{1}{4}$ $\frac{2}{8}$ $\frac{4}{16}$
- $\frac{1}{2}$ $\frac{4}{8}$
- $\frac{2}{3}$ $\frac{4}{6}$

Look closely at the denominators:

- **4, 8, 16**
- **2, 8**
- **3, 6**

If you multiply the numerator of $\frac{5}{6}$ by 3 and the denominator of $\frac{5}{6}$ by 3, have you changed the fraction?

If you multiply only the numerator or denominator, but not both, have you changed the fraction?

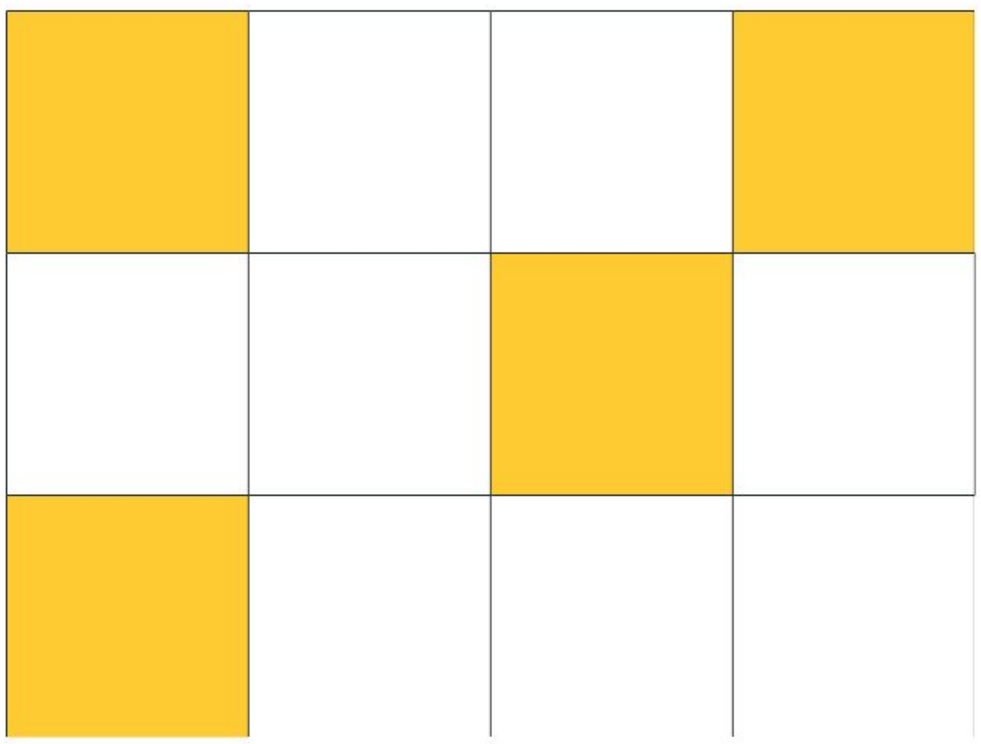
Use diagrams to support your answer.

Why do we simplify?

Have you ever heard anyone say they have seen six-twelfths of a movie? Although $\frac{6}{12}$ and $\frac{1}{2}$ are completely equal, in the real world it is much easier to visualize things when they are in their lowest terms. This is why it is so important to **simplify** (reduce) your final answers. Simplifying takes an abstract number and gives it meaning.

HOW TO SIMPLIFY VISUALLY

The diagram represents Govind's pay in a year. Each square represents one month's salary that he has earned, and the shaded squares represent money he has donated to the local women's shelter. How many squares are shaded in? How many squares are there in total? How could you simplify this drawing?



Now let's look at the numbers. Just as you were able to create equivalent fractions by multiplying the numerator and denominator by the same number, you are also able to go the other way, **dividing** the numerator and denominator by the same number.

Example

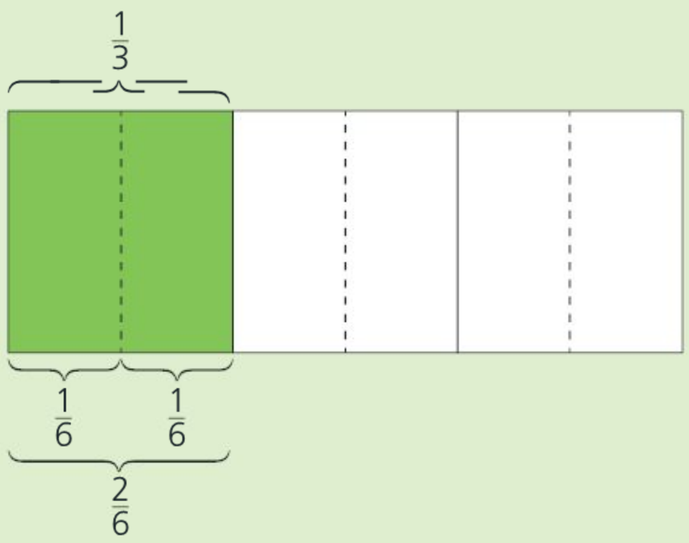
We know $\frac{1}{3} = \frac{2}{6}$:

$$\frac{1}{3} \xrightarrow{\times 2} \frac{2}{6}$$

But if we start with $\frac{2}{6}$, how can we 'go backwards' to $\frac{1}{3}$? How do we know that $\frac{2}{6}$ is equal to $\frac{1}{3}$? What is the reverse of multiplication?

$$\frac{2}{6} \xrightarrow{\div 2} \frac{1}{3}$$

Let's try it visually:

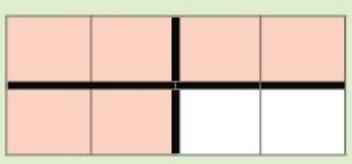


To **simplify a fraction** to its lowest terms, we divide the **numerator and denominator by their highest common factor**.

Now you try. Simplify $\frac{12}{20}$ to its lowest form. First, use a diagram and do it visually, then try dividing numerator and denominator by their highest common factors to simplify numerically.

PRACTICE EXERCISE

1 Simplify:



$\frac{6}{8} =$

2 $\frac{25}{30} \xrightarrow{\div ?} \frac{5}{6}$

3 Simplify:

a $\frac{4}{12}$ **b** $\frac{10}{45}$ **c** $\frac{21}{63}$

4 Simplify:

a $\frac{123}{144}$ **b** $\frac{42}{93}$

When do we need common denominators?

EXTENSION

Why is $\frac{1}{6}$ divided by $\frac{3}{1}$ the same as $\frac{1}{6} \times \frac{3}{1}$? **Use** diagrams to support your answer.

Based on the patterns you have noticed in the activity below, what can you **determine** about common denominators? When are they needed and when are they not?

ACTIVITY: Patterns in numerators and denominators

ATL

- Critical-thinking skills: Draw reasonable conclusions and generalizations
- Transfer skills: Inquire in different contexts to gain a different perspective

Imagine you and some friends split your money equally to buy a lottery ticket. Your ticket wins, and you divide the winnings equally.

1. **If there are four of you in total, what fraction of the winnings do you each receive?**
2. **If there are 10 of you now, what fraction does each person get?**
3. **Which of your answers from Questions 1 and 2 gives the larger amount of money? Draw a diagram to support your response.**
4. **What do you think happens to the size of a fraction as the denominator gets larger (assuming the numerator doesn't change)? Use your own example to support your response.**

EXTENSION

Under what circumstances might it be **unfair** to divide the money equally?

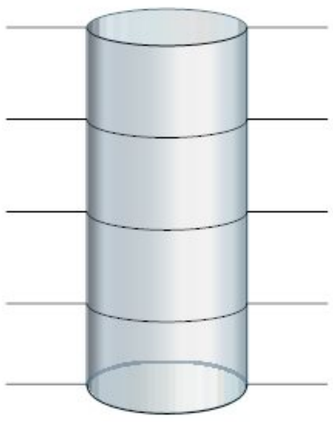
5. **You and four friends have a group project to work on. You do $\frac{2}{5}$ of the work.**
 - a **Is this fair? Why or why not?**
 - b **Why was it easy to compare the fraction of work with the number of people?**
6. **If you ate $\frac{3}{4}$ of a bag of popcorn and the person next to you ate $\frac{4}{5}$ of a bag of popcorn, who had more popcorn? Why is it more difficult to compare these fractions than in the last problem?**

Now answer these questions:

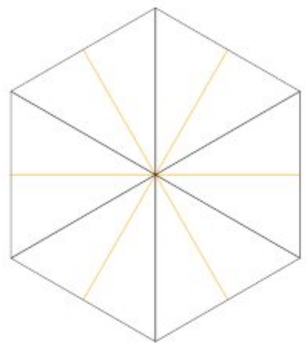
7. a **The shape is divided into thirds. Copy the figure and shade in half of one of the thirds. What fraction of the whole shape does the shaded area represent?**



 - b **This shape is divided into quarters. Copy the figure and shade in half of one quarter. What fraction of the whole shape does the shaded area represent?**



 - c **The next shape is divided into sixths. Copy the figure and shade in half of five of the sixths (leaving one-sixth completely empty). What fraction of the whole shape does the shaded area represent?**



 - d **How do the fractions in each of the problems a to c relate to each other? Without drawing, how might you determine what a third of a quarter might be? Or a quarter of a half? Or a quarter of 20? Can you come up with a general rule about taking 'a fraction of' something?**
 - e **Would your rule change if the numerators were something other than 1? (Think about finding $\frac{2}{3}$ of something.)**

Assessment opportunities

- This activity can be assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

COMMON DENOMINATORS

In order to compare fractions with different numerators and denominators, we need a common denominator. Until we have this, comparing fractions is like comparing Euro and yuan. Which is more: 50 Euro or 42 yuan? We need a common currency in order to understand how these numbers relate to one another. Similarly, we need fractions to have the same denominators to compare them accurately, to order them, and even to add or subtract them.

Finding the LCM and HCF

To find the **lowest** or **least common multiple (LCM)** of two numbers:

1. List a few multiples for each number by skip counting.
2. Identify those that are common to both numbers and pick the lowest one. (This is often referred to as 'the peanut method'.)
 Multiples for 2 and 3:
 2, 4, 6, 8 …
 3, 6, 9, 12 …

To find the **highest** or **greatest common factor (HCF or GCF)**:

1. List all factors for both numbers.
2. Identify those that are common and pick the highest one.
 50: 1, 2, 5, 10, 25, 50
 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

Often when we reduce fractions to their lowest terms, we divide them by a factor they have in common, then divide again and again, until they no longer have any factors in common. For example:

$$\frac{50}{100} = \frac{25}{50} = \frac{5}{10} = \frac{1}{2}$$

If we divide these numbers by the HCF in the first place, we turn this into a one-step process (**divide numerator and denominator** by 50):

$$\frac{50}{100} \overset{\div 50}{=} \frac{1}{2}$$

DISCUSS

When you add 3 Euro (€) to 4 Chinese yuan (¥), you get … wait for it … €3 and ¥4. There is no way to add these until they become one common currency. One or both must change their form. What are some of the ways you could do this?



Euro and yuan

How do we find a common denominator?

Recall this: if you multiply the numerator and denominator by the same number, you get an equivalent fraction. But how do you know what to multiply by?

1 What is the LCM of the two (or more) denominators?

For example, consider $\frac{1}{5}$ and $\frac{2}{3}$. The denominators are 5 and 3; 15 is the LCM.

2 Look at the first fraction. What do you need to multiply the denominator by to reach the LCM?

For example, thinking about $\frac{1}{5}$, 5 × 3 = 15, so multiply the numerator and denominator by 3.

$$\frac{1}{5} \xrightarrow{\times 3} \frac{3}{15}$$

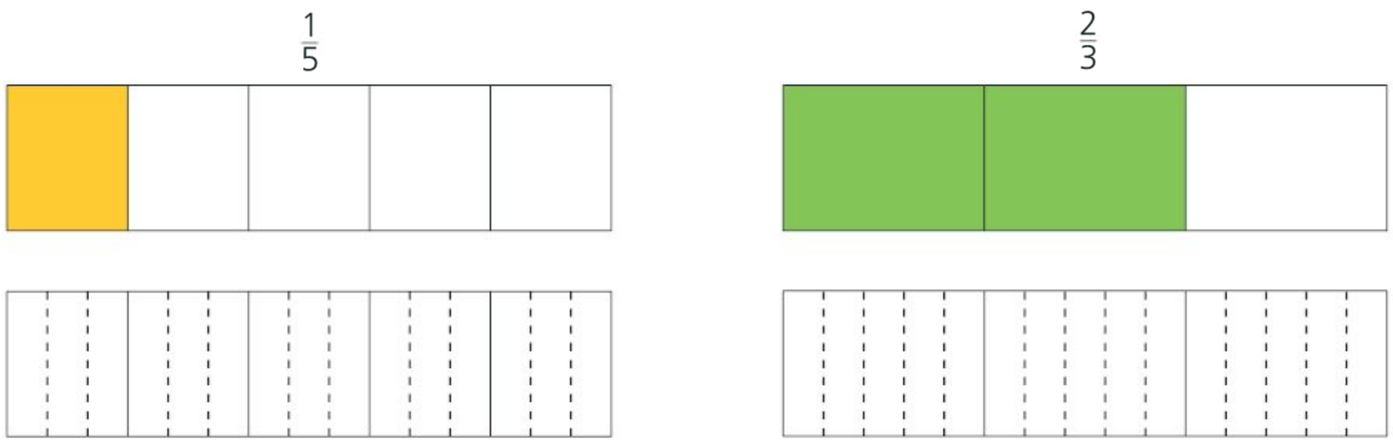
3 Repeat this step for each fraction.

In our example, the second fraction is $\frac{2}{3}$: 3 × 5 = 15, so multiply the numerator and denominator by 5.

$$\frac{2}{3} \xrightarrow{\times 5} \frac{10}{15}$$

4 Now, perform any necessary operations to complete the question (comparison, addition, subtraction).

Visually, this is what has happened:



How do we divide something that was already divided?

ACTIVITY: How much food is going to the food bank?

ATL

- Creative-thinking skills: Use brainstorming and visual diagrams to generate new ideas and inquiries; Apply existing knowledge to generate new ideas

A school is holding a food-bank drive and each of its 12 home rooms has a donation box. At the end of the day, each box is $\frac{3}{8}$ full. The school must combine all the donations into as few boxes as possible to send to the food bank. How many empty boxes will be left? How many boxes of food are going to the food bank? **Solve** this problem using simple drawings and NOT using fraction operations (Communication).

Look closely at your solution. Did your denominator change? Why or why not?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

MULTIPLYING FRACTIONS

We all know that half of 6 is 3, but do we know how to write that mathematically? What happens when we put together 6 groups of $\frac{1}{2}$ (or $6 \times \frac{1}{2}$)?

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{6}{2}$ (adding the numerators since there is already a common denominator)

$\frac{6}{2}$ simplifies to $\frac{3}{1}$ or just 3. This means that when we take a fraction of a number, we are effectively multiplying it. When you see 'of' translate it to 'multiplied by'.

Notice anything else? We just proved that $\frac{1}{2} \times \frac{6}{1} = \frac{6}{2}$. How are the numerators related? Does the same rule apply to the denominators?

PRACTICE EXERCISE

1 Which is larger?

a $\frac{3}{5}$ or $\frac{3}{8}$

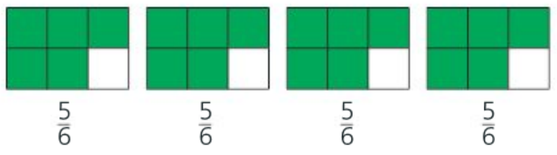
b $\frac{5}{6}$ or $\frac{11}{12}$

2 **List** three fractions equal to $\frac{1}{4}$.

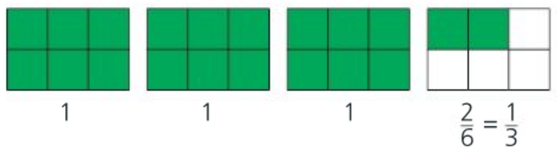
3 A cat notices two glasses of milk on a picnic table. One is $\frac{1}{4}$ full while the other is $\frac{3}{8}$ full. The cat knocks over the glasses and drinks the milk. If she drinks it all, how much of a glass does she consume? If you need to, use a diagram to help you.

4 How did this exercise help you **find** a common denominator? Could you find one without using a diagram? How are the denominators in the activity related to each other?

Let's examine $4 \times \frac{5}{6}$, this time visually:



If we count up the shaded boxes, we have $\frac{20}{6}$, which simplifies to $3\frac{1}{3}$. Or, if you don't like simplifying, you can find the answer by rearranging the shaded squares so the first 20 are shaded, like this:



Cross-reducing

When multiplying fractions with big numbers, we can make our lives a little easier by cross-reducing where possible.

A problem like this takes a lot of time and effort:

$$\frac{18}{25} \times \frac{2}{63}$$

Denominator

$$\begin{array}{r} {}^{3}\overset{1}{2}5 \\ \times\ 63 \\ \hline 75 \\ +\ 1500 \\ \hline 1575 \end{array}$$

Numerator

$$\begin{array}{r} 18 \\ \times\ \ 2 \\ \hline 36 \end{array}$$

so $\frac{18}{25} \times \frac{2}{63} = \frac{36}{1575}$

Wait! Now we need to simplify $\frac{36}{1575}$ … but by what? It isn't clear at first, but the digits in the numerator and the digits in the denominator each add to a multiple of 9. Since that's a divisibility rule for 9, let's try dividing by 9:

$$\frac{36}{1575} \xrightarrow{\div 9} \frac{4}{\quad}$$

$$\frac{36}{1575} \xrightarrow{\div 9} \frac{\quad}{175}$$

hmm … need to do this on the side

$$\begin{array}{r} 175 \\ 9\overline{)1575} \\ -\ 9\downarrow\ \ \ \\ \hline 67 \\ -\ 63\downarrow \\ \hline 45 \\ 45 \\ \hline 0 \end{array}$$

Final answer is $\frac{4}{175}$.

Phew! That was a lot of work! How can we do it more efficiently?

$\frac{18}{25} \times \frac{2}{63}$ ← notice that a number in the numerator and a number in the denominator share a common factor

$\frac{18}{25} \times \frac{2}{63}$

If these numbers were in the same fraction, they would simplify like this:

$$\frac{18}{63} \xrightarrow{\div 9} \frac{2}{7}$$

Guess what? Because this is a multiplication question, we can simplify these numbers, even if they are in two different fractions! Just make sure one number is in the numerator and the other is in the denominator and that they share a common factor.

In summary:

$$\frac{18}{63} \xrightarrow{\div 9} \frac{2}{7}$$

$$\frac{18}{\quad} \times \frac{\quad}{63}$$

$$\frac{18}{25} \times \frac{2}{63}$$

$$\frac{18}{25} \times \frac{2}{63} \quad \div 9$$

$$\frac{2}{25} \times \frac{2}{7}$$

$$\frac{4}{175}$$

DISCUSS

Why is it that the denominators change when multiplying, but stay the same when adding?

MEET A MATHEMATICIAN – MARK ZUCKERBERG (1984–PRESENT)

Learner profile: Caring

We often hear that celebrities and other wealthy people have made large charitable donations. Generous though these donations are, some people say that the amounts won't make much of a dent in the celebrities' fortunes – given the large amount of money they have, they won't even notice that the amount they donated is gone.

Mark Zuckerberg, computer programmer, mathematician and founder of the social networking site Facebook, has pledged to donate 99% of his 45 billion dollars in assets to various charities throughout his lifetime.

Will that alter Zuckerberg's lifestyle? Is this better than him coming up with a fixed number and donating that? Is it fair to judge anyone who makes charitable donations, regardless of the size or amount? Why do people donate money?



Mark Zuckerberg, founder and CEO of Facebook

PRACTICE EXERCISE

Cross-reduce the fractions in the problems below. Don't solve!

$\frac{32}{35} \times \frac{1}{16}$ $\frac{3}{20} \times \frac{15}{16}$ $\frac{12}{13} \times \frac{26}{30}$

Did you notice that in the last question there are **two** opportunities to cross-reduce?

Let's find half of two-thirds (remember, 'of' translates to 'times'):

$\frac{1}{2} \times \frac{2}{3}$

$= 1 \times \frac{2}{2 \times 3}$

$= \frac{2}{6}$ or $\frac{1}{3}$

I multiplied, but my product is smaller than what I started with. Why?

Visually, this is what we just did:



Half of this



$\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$

OR



Divide each third by $\frac{1}{2}$ … now we have sixths. Half of $\frac{4}{6}$ is $\frac{2}{6}$.

And $\frac{2}{6}$ is the same as $\frac{1}{3}$!

EXTENSION

Use the number 1 expressed in a different form to explain why the statement below is true:

$\frac{3}{4} \times 1 = \frac{15}{20}$

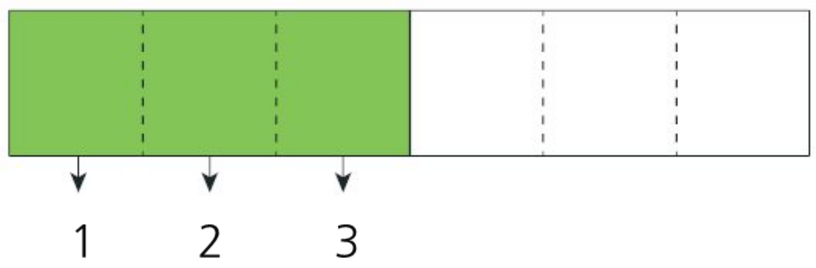
DIVIDING FRACTIONS

How about dividing fractions?

Suppose you have half of a chocolate bar left from yesterday and want to eat the rest in equal amounts over the next three days. In other words, you want to divide that half chocolate bar into three equal parts and you want to eat a third of half a bar each day.



Divide this by 3



Each day you get $\frac{1}{3}$ of $\frac{1}{2}$ of a chocolate bar or $\frac{1}{6}$ of the whole.

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Since $\frac{1}{2} \div 3$ is the same as taking **a third of a half**, we can write:

$\frac{1}{2} \div 3$

$= \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$

In general, we can change a division question into a multiplication question by 'flipping' (taking the reciprocal of) the second fraction. Let's try another one:

$\frac{1}{4} \div \frac{2}{5}$

$= \frac{1}{4} \times \frac{5}{2}$

$= \frac{5}{8}$

Are real-life numbers commonly whole numbers?

ADDING AND SUBTRACTING DECIMALS

Example

Suzette is adding 12.03 + 0.1

$$\begin{array}{r} 1\,2.0\,3 \\ +\ 0.1 \\ \hline 1.3\,0\,3 \end{array}$$

She knows her answer is wrong because her sum is smaller than the number she started with. When we add positives, our numbers should get bigger. What did she do wrong?

Hint

Incorrect alignment of digits is a common error. So, don't line up digits, line up the decimals! That way the tens line up, the units (ones) line up, the tenths line up, and so on.

Solution

Now let's try lining up those decimals properly:

$$\begin{array}{r} 1\,2.0\,3 \\ +\ 0\,0.1\,0 \\ \hline 1\,2.1\,3 \end{array}$$

A **very** different answer from 1.303! Remember, when adding or subtracting decimals, to line up the decimals (include zeros in empty place values if this helps you).

ACTIVITY: Should all basketball nets be regulation height?



ATL

- Critical-thinking skills: Evaluate propositions; Draw reasonable conclusions

According to the USA's National Basketball Association (NBA), the standard basketball net measures 3.028 m in height and the average NBA player is 2.01 m tall. What is the difference between a player's height and the net? Is it fair that we use the same nets in middle schools? Why or why not?

Many basketball fans want to raise the height of the net in professional games. Are these fans justified? If the NBA were to raise the net, what new height would be reasonable? **Justify** your response.

Assessment opportunities

- In this activity, you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

Islwyn drives into a petrol (gas) station and fills his scooter with 5.4 L of petrol. The prices are listed in the table. How much will Islwyn save by purchasing Regular grade petrol as opposed to Premium?

	Regular	Premium
Price per litre (£)	1.38	1.58
Cost of 5.4 L of petrol	5.4 × 1.38	
=	£7.45	

When you have found the products, you can determine what Islwyn has saved by calculating their difference.

MULTIPLYING DECIMALS

Khashi lives in Mongolia and is familiar with the metric system. His mother says he must wear a jacket until it is 15° C outside or higher. If Khashi spends a spring term as an exchange student in Pennsylvania, USA, at what temperature in Fahrenheit may he remove his jacket?

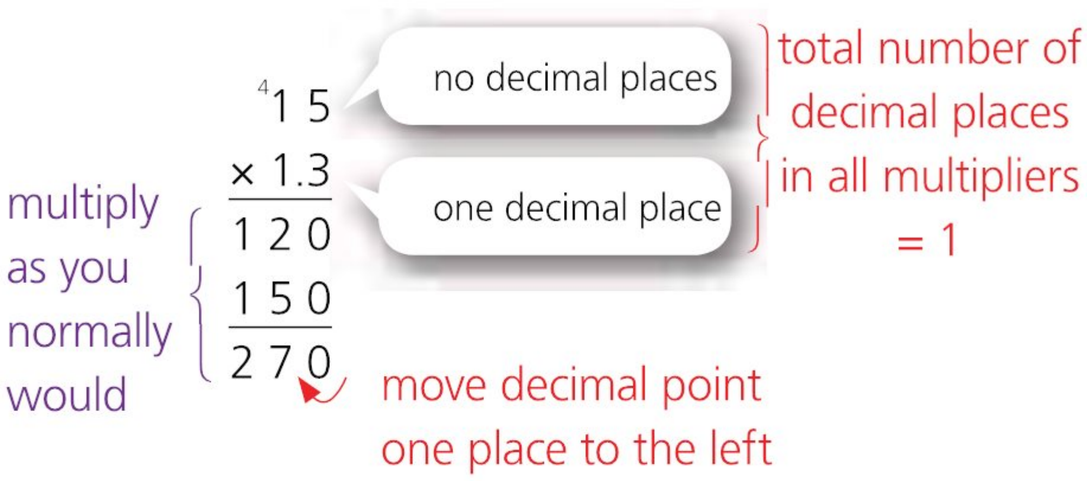
Hint

To convert from Celsius to Fahrenheit, multiply the temperature by 1.8 and add 32.

Khashi's calculation might look like this:

$$(15 \times 1.8) + 32$$

Consider the first part of the calculation: 15 × 1.8. Unlike adding decimals, here we line up the last digits in each number and ignore the decimal point.



$$\begin{array}{r} {}^{4}15 \\ \times\ 1.3 \\ \hline 120 \\ 150 \\ \hline 270 \end{array}$$

So, $(15 \times 1.8) + 32$

$= 27.0 + 32$

$= 59$

Khashi may remove his jacket when the temperature reaches 59°F.

DIVIDING DECIMALS

A common use of division is to compare prices when buying things in multi-packs. Prices calculated in this situation rarely work out to whole numbers, so let's take a look at division with decimals.

Penelope needs to buy the least expensive pack of diapers for her sister Cora. Which pack should she buy?

Diaper package 1

4 × Packs of 26 = 104 Diapers

$15.20



Diaper Package 2

72 Diapers

$9.99



Are these prices fair? Why might one type of diaper be more expensive than the other? Are the products of equal value?

Let's see what each package costs **per diaper**:

	Package 1	Package 2
Cost per diaper ($)	15.20 ÷ 104	9.99 ÷ 72
=		

Package 1

$$\begin{array}{r} 0.146 \\ 104\overline{)\,15.200} \\ -104\downarrow\times\times \\ \hline 480 \\ -416\downarrow\times \\ \hline 640 \\ -624 \\ \hline 16 \end{array}$$

Package 2

$$\begin{array}{r} 0.138 \\ 72\overline{)\,9.990} \\ -72\downarrow\times\times \\ \hline 279 \\ -216\downarrow\times \\ \hline 630 \\ -576 \\ \hline 54 \end{array}$$

Since we are dealing with money (in this case in dollars and cents), we can round to the nearest hundredth (or 2 decimal places):

- Package 1 works out at $0.15 per diaper.
- Package 2 costs $0.14 per diaper.

Therefore, Package 2 is the cheaper of the two.

In the diaper calculations, the **divisor** was a whole number. But what if we are dividing by a decimal?

THINK-PAIR-SHARE

1 **Write $\frac{1}{3}$ as a decimal.**

Hint

What is 1 divided by 3?

2 **Now, take that number and add it three times:**

your number + your number + your number

But …

3 **What is $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$? (Imagine filling a measuring cup with $\frac{1}{3}$ cup of flour three times.)**

It doesn't make sense! In the case of a recurring (never-ending) decimal, which is more accurate, the fraction or the decimal?

Kaito receives a bonus at work if he sells over 110 candies from the 10.88¥ (Japanese Yen) range, but he lost track of how many candies he'd sold throughout the day! He counted 1360.00¥ in the cash register – all sales from the candies. Did Kaito earn his bonus?

To determine the number of candies sold, Kaito needs to divide the total amount by the candy price: 1360¥ ÷ 10.88¥.

How do we divide this?

1 Multiply the divisor by a power of 10 (that is, by 10, 100, 1000, etc.) to make it a whole number.
So in Kaito's example:
- 10.88 × 10 = 108.8 – still not whole!
- 10.88 × 100 = 1088 – there we go!

2 Multiply the dividend by the same power of 10:
1360 × 100 = 136000.00

3 Place the decimal point in the quotient directly above the decimal point in the dividend and divide everything the way you would if they were whole numbers to begin with.

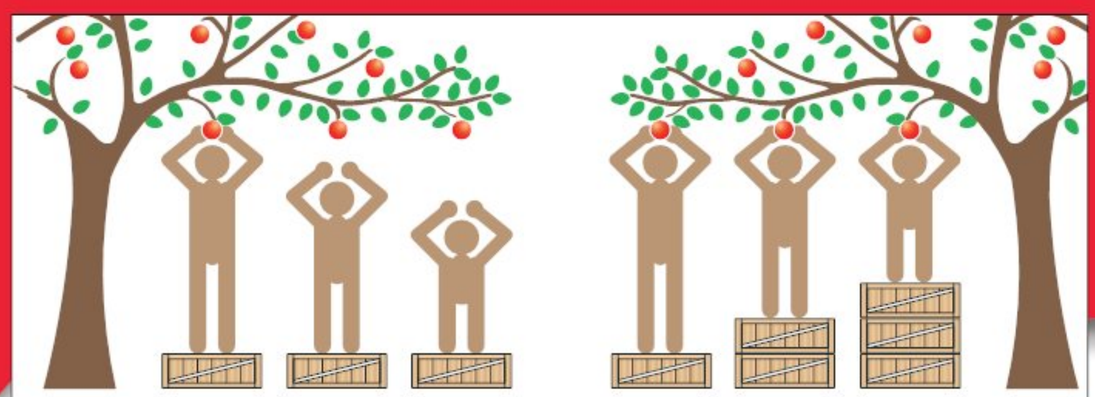
$$\begin{array}{r} 125 \\ 1088\overline{)\,136000.00} \\ -1088 \\ \hline 2720 \end{array}$$

$$\begin{array}{r} 125 \\ 1088\overline{)\,136000.00} \\ -1088\downarrow\times \\ \hline 2720 \\ -2176 \\ \hline 5440 \end{array}$$

$$\begin{array}{r} 125.00 \\ 1088\overline{)\,136000.00} \\ -1088\downarrow\times\times \\ \hline 2720 \\ -2176\downarrow \\ \hline 5440 \\ -5440 \\ \hline 0 \end{array}$$

So, Kaito sold 125 candies from the 10.88¥ range; he earned his bonus!

So, is fairness always equal?



DISCUSS

Let's reflect further on Mark Zuckerberg's donations (see page 21). If a person earning $80 000 per year donated $20 000, and a person earning $5 000 000 a year donated the same, they are making **equal** contributions – but one is sacrificing more than the other. Why?

Can you think of other real-world examples in which it is more difficult for one person to achieve an equal quantity compared to another? Perhaps in economics? Sports? Education? Other thoughts?

What does 'per cent' really mean?

Of the three forms of proportion commonly used, probably the most common is **percentage**. Consider these price discounts:

- discount of 0.1 times the price
- $\frac{1}{10}$ off
- 10% off.

These expressions all refer to the same discount, but the last is the easiest to say and visualize.

Shade in 80 hundredths of a circle. Now try shading in 80% of a circle. Which comes more naturally?

'Ten per cent off a price' in Euro means 'for every hundred Euro, you get 10 off'. But the item may not cost exactly a hundred Euro, so we adjust using proportions. Percentages are proportions of 100.

WHAT IS A PROPORTION?

DISCUSS

If we want to crop out the photobomber in the minimized picture below and blow it up to fit in a regular 4 × 6 inch frame, why would the cropping shown **not** work?





Changing two or more related values – be they picture lengths, dimensions in a model building, ingredients in a recipe, or simply numbers in a fraction – and ensuring their sizes change without changing their relationship is known as **maintaining proportion**.

Change the percentage to a decimal by dividing by 100, and then multiply the total you are given to find the proportion.

Divide the portion you are given by total and multiply by 100.

Finding a portion, given the percentage

Example

If the owner of a franchise restaurant is allowed to keep 15% of all sales (and the rest goes to the franchise), how much would he earn from the sale of a $8.75 meal?

Solution

$\frac{15}{100}$ = 0.15 (convert percentage to decimal by dividing by 100)

0.15 × 8.75 = 1.3125 (multiply decimal by total to find proportion)

Thus, the owner earns $1.31 (we round to the nearest hundredth since we are dealing with money) from each sale of $8.75.

Finding the percentage, given a portion

Some quizzes are marked as a number of correct answers out of a total of possible answers. This proportion of marks is less meaningful than the equivalent percentage. To work it out, simply divide the numerator by the denominator (the **portion** of correct answers by the **total** number of answers) then multiply by 100.

Example

Did James score above 65% if he got $\frac{17}{28}$?

Solution

17 divided by 28 = 0.6071 (rounded)

0.6071 × 100 = 60.71%

No, James did not score above 65%.

EXTENSION

Franchises are businesses that are owned by individuals but have a parent company that allows them to use the company name and brand for a fee.

If the owner of one franchise earns $350 000 in monthly sales, and the owner of another franchise in the same chain with the same expenses earns $640 000, is it fair that they both owe an equal amount – $300 000 – to the parent company?

ACTIVITY: Chandra's mobile phone storage

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations

Chandra pays for 3GB of data on her mobile phone every month, and has to pay extra if she exceeds this limit. Her phone keeps track of her data usage throughout the month.

1. **If she has used 13% of her data by the fourth day of the month, how many GB of data is this?**
2. **If she uses 1.7GB, what percentage of her data limit has she used?**
3. **If she uses an average of 0.1GB per day, what per cent of her data will she have used by the twentieth day of the month? Will she go over if she continues using data at the same rate?**
4. **Is it possible for Chandra to use 125% of her data allowance? What does this mean?**

Assessment opportunities

- In this activity, you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

Do different forms lead to different solutions?

WHICH WAY OF EXPRESSING PROPORTIONS IS THE BEST?

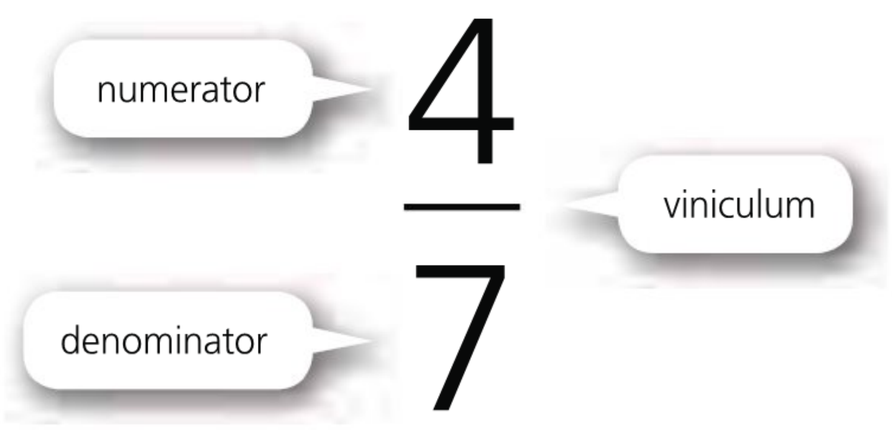
Converting from fractions to decimals

We know what the terms numerator and denominator mean in fractions, but what makes a fraction a fraction and not a time signature in music notation, for example? It is the **vinculum** – the bar that separates the numerator from the denominator.

Essentially, a viniculum is another **form** of a division symbol (÷). So, $\frac{1}{2}$ simply means 'one divided by two', and if you use long division to divide one by two, you will get 0.5. Thus, to **convert from a fraction to a decimal, divide the numerator by the denominator**!

Converting percentages to decimals, and vice versa

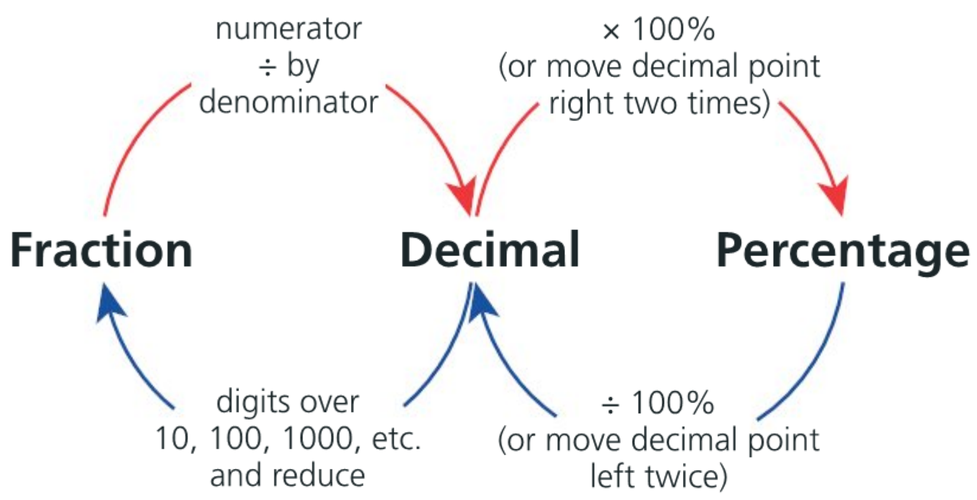
Remembering that 'per cent' literally means 'for every hundred' can help remind you how to convert percentages: **multiply the decimal by a hundred, and then include the percentage symbol**. To convert from a percentage to a decimal, we simply reverse this – **divide the percentage by a hundred and remove the percentage symbol**.



How does place value help to convert decimals to fractions?

Remember that, for instance, a number such as 0.45 is read as '45 hundredths'. This directly translates to $\frac{45}{100}$. To convert from a decimal to a fraction, **translate the decimal from place value terminology to a fraction** (over 10, 100, 1000, etc.) and then simplify.

When changing numbers from one form to another – be they units of measurement, currencies or proportions – it is helpful to use a conversion flow chart. These charts often have arrows pointing in opposite directions from one form to another, with instructions on the arrows to describe how to convert from the first to the second, and back again. This conversion flow chart describes how to change forms from fractions, to decimals, to percentages and back again.



Now you try! Create a conversion flow chart that shows how to convert a fraction to an equivalent but larger form, and how to simplify it back again.

SOME SUMMATIVE PROBLEMS TO TRY

Use these problems to apply and extend your learning in this chapter. The problems are designed so that you can evaluate your learning at different levels of achievement in Criterion A: Knowing and understanding.

THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 1–2

1 Which of the following are integers?

- 3
- –8.9
- 1
- 7
- –4
- 1.2
- $-3\frac{1}{2}$

2 **Draw** a number line from –10 to 10 and **label** all of the numbers from question 1.

3 **Simplify**:

a –1 – 3
b –5 + 6
c 3 – 7
d 4 – 2 × 6
e $\frac{1}{5} + \frac{3}{5}$
f $\frac{10}{15}$
g $\frac{2}{3} \times \frac{1}{4}$
h 1.2 + 7.9
i 3.1 × 8
j 4.2 divided by 6

4 Circle the operation you would perform first. **Don't simplify!**

a $3 + 5^2$
b $(3 + 5)^2$
c 2 × (5–1)

5 Write 0.62 as a percentage.

6 Which is bigger?

a –5 or 5
b 2 or –100
c $\frac{1}{5}$ or $\frac{1}{6}$
d $\frac{3}{4}$ or $\frac{7}{4}$

THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 3–4

7 Can a number belong to more than one system? **Justify** your response.

8 **Simplify**:

a –(–7)
b –13 + 25
c 49 – (–3)
d 65–102
e $3 + 5^2$
f $(3 + 5)^2$
g $(1 + 4)^2 \times 2$
h $18 \div 3^2$
i $\frac{1}{6} + \frac{1}{3}$
j $\frac{4}{5} - \frac{3}{4}$
k $\frac{6}{7} \times \frac{3}{5}$
l $\frac{1}{9} \div \frac{2}{3}$
m 0.2 + 11.25
n 153.8 – 46.91
o 17.62 × 0.1
p 8.97 ÷ 3
q 4.3 ÷ 12
r 7 ÷ 0.19

9 **Find**:

a $\frac{1}{2}$ of 374
b $\frac{2}{3}$ of 98
c 71% of 86

10 Enhance the communication of this fraction by converting to a mixed number: $\frac{10}{4}$.

11 Copy and complete the table below:

Fraction	Decimal	Per cent
$\frac{11}{16}$		
	0.53	
		84
$1\frac{2}{5}$		

THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 5–6

12 Simplify:

a $2\frac{3}{4} \div 1\frac{1}{2}$
b $\frac{4}{123} \times \frac{3}{28}$
c $1 - 4^2$
d $\frac{45.3}{(12.06 - 9.06)}$
e $10 \times (-9 + 2^2)$
f $\frac{2}{5} \times (\frac{1}{3} - \frac{1}{4})$
g $(\frac{2}{3})^2$
h $-4 + (-6) + \sqrt{49}$

13 Jolene goes to bed three hours after dinner. Yesterday, after dinner, she spent $1\frac{1}{2}$ hours on her homework and $\frac{2}{3}$ of an hour on the telephone. How much time did she have left before bedtime?

14 Which animal had to travel further to get their fish? (**Justify** using integers.)

a An eagle flying 33 m **above** sea level catching a fish 63 m **below** sea level.
b A whale swimming 289 m **below** sea level diving down to a fish 365 m **below** sea level.

15 Dubaku received an employee discount at her store and only paid $1055.99 for a tablet with a selling price of $1199.99. What percentage do employees get to take off of their purchases?

16 Convert $\frac{5}{6}$ to a percentage.

THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 7–8

17 Which is greater: $\frac{1}{3}$ or 0.3? **Justify** your response.

18 Can you take the square root of –9 ($\sqrt{-9}$)? If yes, what is it? If no, why not?

19 Add brackets to make the equation below true:
$30 - 5 \times 2^3 - 15 = 5$

20 Shop-owner Anatole is selling his sweaters for $45.96, but tries to bargain with you and tells you that he will take 20% off his price if you buy it from him. Shop-owner Sergei is also selling sweaters and tells you that he will sell you a sweater for 25% off his regular price of $46.78 if you buy it from him.

How much will you save if you take the better deal? Support your answer by comparing both sale prices.

21 In the reality television series 'The Biggest Loser' contestants who lost the most weight in a given period of time won a cash prize. After a few seasons, they introduced percentage weight loss as the competition goal to overcome any unfair advantage. Did this make it fairer? Do some contestants still have an unfair advantage?

! Take action

! There is a responsibility that comes with being a consumer. Every dollar you spend counts as a 'vote', showing how you would like the economy to run. In 2006, a very popular clothing company in the US was reported to burn defective items rather than giving them to charity, stating 'only people of a certain stature are able to wear the company name'. On the other hand, an equally popular shoe company, Toms, stands by the slogan 'one for one', referring to the pair of shoes they donate for each pair sold.

! There are many companies which donate a percentage of their profits, a percentage of their products, or a fixed amount of money to those in need. These companies range from businesses in the food industry, like Love Crunch, to manufacturers of baby clothes, such as Rockin' Baby.

! However, some members of the public are unaware of how their consumer choices affect global communities. Wouldn't it be nice always to know where your money is going?

! Your task is to **create** a poster or other ad (perhaps a graphic on the school website) to raise awareness in your school about a company that has committed to sharing their profits with those in need.

! In your campaign, be sure to include specifics:

- Is the donation a percentage? If so, how much money would that be from a single purchase and how much on a larger scale (say, on sales of $2 million)?
- What are the alternative choices for the consumer?
- How do you know this company stands behind its word? Perhaps there is something in the media about it?

! Research two other companies and compare their donation methods to **justify** which is the most generous.

▼ Links to: Visual arts – media and marketing

Making a poster allows you to use your creative side. Think about what you have learned about effective communication in the visual arts.

Reflection

Use this table to reflect on your own learning in this chapter.						
Questions we asked	Answers we found	Any further questions now?				
Factual: How do we express quantities? What forms can we use to visualize proportions/parts? When do we need common denominators?						
Conceptual: Does order matter? Can you ever have less than nothing? How do we divide something that was already divided? Do different forms lead to different solutions?						
Debatable: Can inequality ever be fair? Are real-life numbers commonly whole numbers? Is there really such a thing as subtraction? Or multiplication?						
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?				
		Novice	Learner	Practitioner	Expert	
Critical-thinking skills						
Creative-thinking skills						
Transfer skills						
Learner profile attribute(s)	Reflect on the importance of being caring for your learning in this chapter.					
Caring						

2 How can we bring things together?

Identifying and using **patterns** and rules is the key to **simplifying** **relationships**, in life and in algebra.



CONSIDER THESE QUESTIONS:

Factual: What is algebra? What are like terms and how do we collect them? How can I tell if terms are like or not? What are algebraic products?

Conceptual: What is a relationship? How do we express ourselves in algebra? What are unlike terms and what do I do with them? What happens if we have a negative term, or more than one term? Are variables the key to understanding relationships?

Debatable: Why do we need to have common rules of communication? Does it help to be the same or is it better to be different? Could algebra be a universal language?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- the difference between letters and numbers
- how to add, subtract, multiply and divide numbers
- how to recognize and use directed numbers and integers.

IN THIS CHAPTER, WE WILL …

- **Find out** how learning algebra is like learning a new, and useful, language.
- **Explore** the rules of algebra simplification.
- **Take action** by promoting a mathematician who shares some of our identities.

These Approaches to Learning (ATL) skills will be useful …

- Transfer skills
- Affective skills
- Information literacy skills
- Communication skills

Assessment opportunities in this chapter:

- **Criterion A**: Knowing and understanding
- **Criterion B**: Investigating patterns
- **Criterion C**: Communicating
- **Criterion D**: Applying mathematics in real-life contexts

We will reflect on these learner profile attributes …

- **Inquirer** – We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.
- **Communicator** – We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

KEY WORDS

collecting | like | unlike

RELATIONSHIPS

Relationships in MYP Mathematics refers to the connections between quantities, properties or concepts, and these connections may be expressed as models, rules or statements.

Relationships provide opportunities for you to explore patterns in the world around you.



SEE-THINK-WONDER

Look at the mathematical cartoon. Think about why it might be funny. Do you think it is? Why or why not? What does it make you wonder about mathematical jokes in general? Do you know a funny mathematics joke or can you make one up?

What is algebra?



Algebra is the ability to manipulate symbols so that we can understand relationships. It is a body of mathematical knowledge which allows us to find **unknowns**, simplify them and, hopefully, use them to solve problems. Algebra is practising simplicity.

Humans long to solve problems and investigate unknowns. We look for patterns in our lives, reasons to explain or connect events so we can better understand them and predict the future. Making things simpler can help us to bring order to chaos. In fact, it has been said that the brain can be thought of as a pattern-processing machine, because humans are constantly trying to make sense of the universe.



- People in many walks of life look for patterns to solve problems

WHY DO WE NEED SYMBOLS?

Algebraic techniques allow us to use the mathematical methods we already know from arithmetic and **apply** them to new, variable, changeable or unknown circumstances.

We often use letters or symbols to represent **variables** (quantities that can change). Greek letters are commonly used in algebra, as well as in scientific formulae.

What is the difference between:

- a letter and a number
- a letter and a symbol
- a number and a symbol?

You will probably use the Roman alphabet (which is used for English and which you are reading right now), as well as Greek letters in your algebraic learning. This means you can have more than the 26 Roman letters and helps to avoid possible confusion between, for example, x (the variable) and ×, the sign for multiplication. Using Greek letters also reminds us of Ancient Greek mathematicians who made so many contributions to mathematics.

Greek symbols

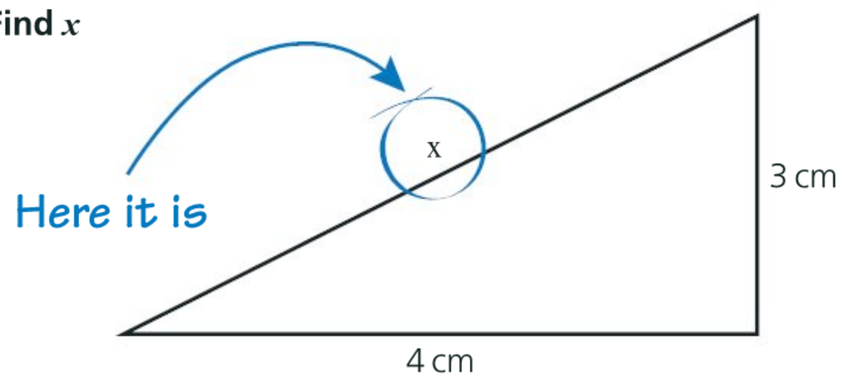
Why do we use Greek symbols? Do you recognize any of these symbols? Where from?

Αα	Alpha	Ιι	Iota	Ρρ	Rho
Ββ	Beta	Κκ	Kappa	Σσς	Sigma
Γγ	Gamma	Λλ	Lambda	Ττ	Tau
Δδ	Delta	Μμ	Mu	Υυ	Upsilon
Εε	Epsilon	Νν	Nu	Φφ	Phi
Ζζ	Zeta	Ξξ	Xi	Χχ	Chi
Ηη	Eta	Οο	Omicron	Ψψ	Psi
Θθ	Theta	Ππ	Pi	Ωω	Omega

Despite the fact that we can choose from so many letters, the most popular letter used in algebra is the letter x. Why is this? Why do you think the second most popular letter in algebra is y?

What does find x really mean?

Find x



ACTIVITY: Why do we use x so much?

ATL

- Media literacy skills: Interacting with media to use and create ideas and information

Things usually happen for a reason. This is as true in mathematics as it is in life! If x is the letter that is most commonly used to show an unknown, there is likely to be a good reason why.

Watch this Ted Talk 'Why Do We Solve for x?':

www.npr.org/2015/03/06/388518850/why-do-we-solve-for-x

Now in your own words, write an explanation of why we often use x to represent an unknown quantity.

What does 'x' mean in a phrase like 'they have the x-factor', 'X marks the spot' or 'I need to get an x-ray on my arm'? How does this relate to the explanation you have just written? Can you make connections between these ideas?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

VARIABLES, UNKNOWNS, CONSTANTS AND TERMS

A **variable** is something which can vary or change in value, like the time of a tide or the price of a house.

An unknown is something you are looking for; you can find this by solving. See Chapter 4 for more on solving.

A **constant** is a term which doesn't change, such as 7 or –2.5. Constants are always real numbers, both positive and negative, with values that are always exactly the same. Consider, 3 is a 3, is a 3, never any other value!

A **term** can be a variable, a constant or a combination of both. Terms are separated from one another by a + or a – sign.

Here is another way to think about algebra: it is a mathematical language that anyone can understand, no matter where they come from. It is similar to music. Anyone can read and play from a musical manuscript, if they have studied musical notation.



ACTIVITY: What changes and what doesn't?

ATL

- Communication skills: Read critically and for comprehension

Some symbols represent variables, while others can represent constants. With your partner, decide which of these are **constants** and which are **variables**:

- **your age**
- **the boiling point of water**
- **gravitational pull**
- **your IQ (intelligence quotient)**
- **the price of electricity**
- **the download speed of your network**
- **the number of subjects you study per year**
- **the price of a train ticket**
- **the length of journey to school**
- **the length of your hair**
- **the name of your mathematics teacher.**

Did you and your partner disagree on any of the answers? Are there some which might change between constant and variable but only under very, very specific conditions?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

How do we express ourselves in algebra?



When eating a bag of candies or sweets, people often eat them differently. Some devour them randomly, while others prefer to sort them by colour before enjoying them. Why would they do this? What is it about organizing candies in a certain way that appeals to them?

Take the multi-coloured sweets in the pictures above. We can see that there were green sweets, yellow sweets, purple sweets, orange sweets and red sweets.

This array of colours could be listed: green, yellow, purple, orange and red.

As we have learned already, algebra uses symbols or letters to represent quantities. The number of sweets or candies of each colour could be represented as follows: g and y and p and o and r and …

Any bag of similar sweets or candies could be represented as follows:

$$g + y + p + o + r$$

You can see how this is already much simpler than the longer list of colours.

You could use algebra to ask (and answer) the following questions about candies:

1. What does each letter represent?
2. How could I find out the value for g for any bag of sweets?
3. Would the value of g change for a different bag?
4. Why is there no letter b in the expression?
5. If you knew values for each letter, what would their total tell you?

Watch out! Remember that a letter such as g for green represents the **number** of green sweets, not just g for green sweets. This is a common communication error. If you see four green sweets, this doesn't mean $4g$. It actually tells you:

$$g = 4$$

That is, the **number** of green sweets equals four.

MYP CRITERION C: COMMUNICATING

At the end of year 1, students should be able to:

i. **use** appropriate mathematical language (notation, symbols and terminology) in both oral and written statements

ii. **use** different forms of mathematical representation to present information.



In the cartoon, you can see two strands of the communication criterion for Year 1 MYP Mathematics. Algebra is the perfect way to practise and use them regularly. You will be using symbols and words, often moving or 'translating' between them. In fact, by the end of year 1 you might even be a fluent algebra speaker!

As with learning any language, we will start with the 'letters and words' (symbols) and the rules that join them. We will work up to short phrases or **expressions** before tying them together into longer sentences or **equations**. The ultimate goal is to be able to write long and elegant 'stories' in algebraic notation.

ACTIVITY: Smartphone toolbar

ATL

- Information literacy skills: Understand and use technology systems

Samir has a smartphone which can be loaded with all kinds of music, photos and apps. The data storage capacity of his phone is colour-coded blue for pictures, orange for apps and green for music.

When he plugs in his phone, he sees the following toolbar. He doesn't know how much space each type of media takes up.

1 **Choose five different letters, one to represent each type of data usage on Samir's smartphone.**

2 **Give a key, or explanation, to say what each letter represents.**

3 **Write an expression to show all the different types of files put together.**

4 **According to the toolbar diagram, how much free space is on the phone?**

5 **If you know that the phone has 16 GB memory space in total, what can you say about your answer to question 3?**

6 **If the phone actually had 8 GB of memory storage, how would it change what you can say about your answer to question 3?**

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.



Note that, in the case of the sweet example, because we are talking about the total or sum of a situation we use a + sign between the terms.

Without really noticing it, you have been creating phrases with multiple terms, as if you were learning words and then you suddenly started putting them together to make short phrases. An expression in algebra is a grouping or phrase of algebraic terms, which might include one or several variables and constants.

Examples of expressions are:

- $4c + 1$
- $3x - 1$
- $a + b + c + d$

Notice that, although we have been using letters here that match the objects, this isn't the purpose of the symbols. They don't represent the objects; they are variables. Variables can be used and applied in different ways. For example, think back to the coloured candy: if each coloured candy cost a different amount to produce, we could use the expression $g + y + p + o + r$ in an equation to calculate costs. This time each letter or symbol represents the **price** of a particular colour of candy, as opposed to the **quantity** of that colour candy, which it represented previously.

COEFFICIENTS

All terms have a **coefficient**. It just so happens that the simplest way to write $1c$ is just c. Not writing 1 doesn't affect the value, just as saying '1 × 5' is the same as just '5'.

Likewise $-j$ has the coefficient –1; the '1' doesn't need to be written, but the minus sign **absolutely** has to be there. Leaving it out changes the value entirely. This would affect your communication (and it would make your answer or working wrong!)

Why do you think we can make the following statement: $1a = a$ or in words, '$1a$ can be written as a'?

Consider this before you answer. Think about how you speak about objects in English. You **could** say 'Can I borrow one pencil?', but you **normally** say 'Can I borrow **a** pencil?' Once you've mentioned 'a pencil', it is assumed you want only one. Otherwise you would have asked for more!

PRACTICE EXERCISE

Circle the coefficient of each of these terms.

- **$14d$**
- **$11k$**
- **$5.3e$**
- **$918\,273k$**
- **$-5g$**
- **$3.5y$**
- **$\frac{2}{3}s$**
- **w**

How can I tell if terms are like or not?

Links to: Language and literature



'You can't compare apples and oranges' is used to remind people that comparison of some things is unfair, inappropriate or impossible. This phrase also applies in algebra when we are talking about like and unlike terms. We can add the apples and we can add the oranges, but we can't add them to each other!

LIKE TERMS

'Collecting **like terms**' is grouping like terms together so you can simply add or subtract their coefficients. 'Like terms' are defined as **multiples of the same symbol**.

2 , 3 and 24 are all like terms because they are all various amounts of . 5 , 1 and –8 are also all like terms.

6 and 6 are **not** like terms, there just happen to be the same number (six) of each of the two terms.

ACTIVITY: Like the likes

ATL

- Communication skills: Understand and use mathematical notation

Copy and complete by shading the like symbol for like terms from each row in the table.

1	4	2x	3	11
2	1	–4	–4	12.5
3	3	–2	3	15
4	4	2	2 ×	15
5				

Now create a row of your own. Make sure that there are some like terms (but not more than one type because it might get confusing).

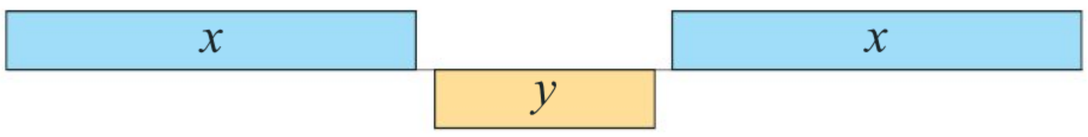
Now swap your row with a friend and solve each other's rows.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

WHEN TERMS ARE DIFFERENT

From what we know about **like terms**, we can **deduce** that **unlike terms** must be terms which are **not** multiples of the same symbol, such as , and . , 3 and –2 are like terms, while , and – ^{2} are not. Why is this? In this diagram, x and x are clearly like terms, while is unlike or different.



HOW DO WE COLLECT LIKE TERMS?

Adding multiples of the same terms in algebra uses the same process as adding numbers in arithmetic. Look at these examples.

+ + = 3

3 + 7 + 11 = 21

$\frac{2}{3}$ + $\frac{4}{3}$ + 2 + 1 = 5

If we consider that + + is the same as 'three times ', then technically any of the following would be correct:

- 3 ×
- 3
- 3()

From the examples above, you will see that these expressions can also be written simply as 3 . Make sure this is the form you use in your answers to demonstrate excellent **communication**.

PRACTICE EXERCISE

Practise collecting like terms by finding:

1 a + a =
2 2c + c + c =
3 10e + 23e – 11e =
4 2g – 0.5g – 0.5g =
5 88i + 101i + 3i =
6 –600k + 412k =

SIMPLIFYING ALGEBRAIC EXPRESSIONS BY ADDITION AND SUBTRACTION

Simplifying fractions means reducing them to their lowest, easiest-to-understand terms. Simplifying algebraic expressions does the same job, by collecting all like terms and summarizing them.

There are a few different ways to visualize the collection of like terms. Choose your favourite method and visualize this when you are simplifying. You don't *need* to show which method you prefer, but it will strengthen your algebraic reasoning for more complex work later.

Using colour or order to collect

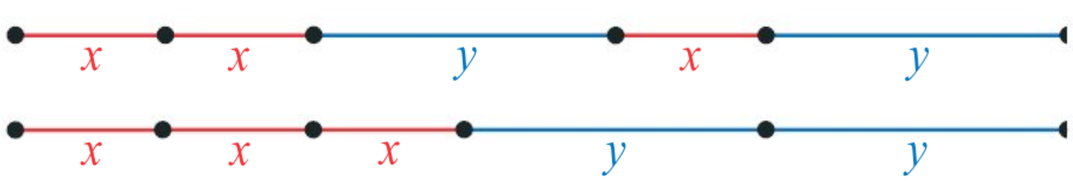
$x + x + y + x + y$

$x + x + x + y + y$	rearrange so that like terms are all together
$3x + y + y$	add all the x like terms
$3x + 2y$	add all the y like terms

Using diagrams to collect

$x + x + y + x + y$

Here we are using a number line to collect like terms.



Striking through

$x + x + y + x + y$

$\cancel{x} + \cancel{x} + y + \cancel{x} + y$

$3x + y + y$	now you have collected the x like terms
$3x + 2y$	now you have collected the y like terms

Sometimes students make the following mistake:

$x + x + y + x + y$

$3x + 2y$

$5xy$

Answer = $5xy$

This is called **conjoining** terms. But it is **wrong** to add or subtract terms in this way. Remember x and y are unlike terms and so can't be combined!

WHY ARE CONJOINING TERMS INCORRECT?

$5xy$ is actually $xy + xy + xy + xy + xy$, which is a very different answer from $3x + 2y$!

Still don't believe conjoining is wrong? Let's put some number values in these expressions – a technique called **substitution**. Let's say instead of x we have 1 and y is actually 2.

$3 \times 1 = 3$ and $2 \times 2 = 4$

So, the values of $3x$ and $2y$ would add up to 7.

But $5 \times (1) \times (2) = 10$

So the value of 5 multiplied by x and by y ($5xy$) equals 10. The expressions are clearly **not** the same, as 7 doesn't equal 10. One must be right ($3x + 2y$) and one is wrong ($5xy$)!

You will learn more about **substitution** in Chapter 4.

PICTURES AS LIKE TERMS

Let's think about pictures as terms to see if we can improve our understanding of like terms.

Example

Simplify

● + ⟦ + ⟦ + ● + ⟦ = 2● + 3⟦

This is the final answer as we clearly can't add ● to ⟦ in any real or sensible way.

PRACTICE EXERCISE

Simplify:

1 $b + c + c + b$	**4** $\square + \blacksquare + (\blacksquare + \blacksquare) - \square$	**7** $17t + 4t + 2u - u + t =$
2 $3t + t + u$	**5** $r + r + r + r + r =$	**8** $a + b + c + a + b + c =$
3 $\bigstar + \bigcirc + \bigcirc + 2\bigstar$	**6** $10s - s - s - s =$	
9 $10v + 11v + 3w + 2w$	**12** $ab + bc + ab + 3bc$	**15** $0.1i + 0.2j + 0.3i + 0.9j =$
10 $6x + 0.5x + y + 2.5y$	**13** $2v + w + 3w - 2v =$	**16** $kl + 2l + 7l + 3k + lk$
11 $\frac{3}{2}z + 2z + 4a + \frac{5}{4}a$	**14** $10x + 11x + 12y + 13y + 14x =$	
17 $-5d - 3e + 7e - 6d$	**20** $62q + 108q + 4q - 2q^2$	**23** $\frac{3}{2}t + \frac{1}{3}u + \frac{2}{6}t - \frac{4}{5}u$
18 $f(g) + 2gf - fg + fgh$	**21** $-2m - n - 4m$	**24** $(3 + 4)a - (-2a + 4a)$
19 $\varphi + 3 + \varphi + \varphi + 1$	**22** $p + q + r + pq + qp + pr$	

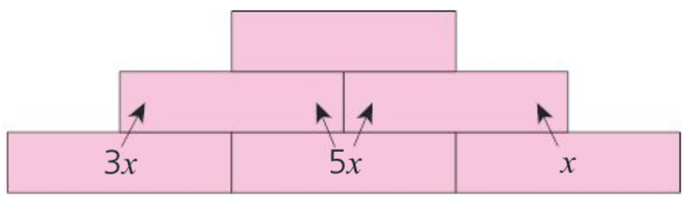
ACTIVITY: Race to the top

ATL

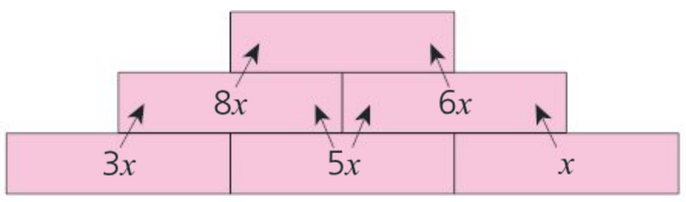
- Transfer skills: Apply skills and knowledge in unfamiliar situations

Collecting like terms in a pyramid

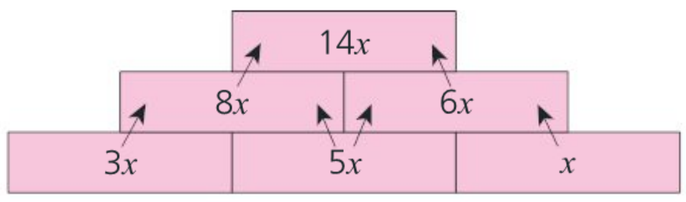
Step 1: Start at the bottom row and **add** adjoining (touching) boxes together to give the box directly above them.



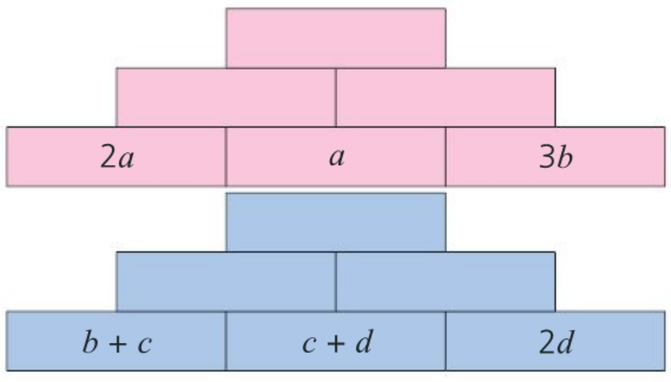
Step 2: Repeat for the next row.

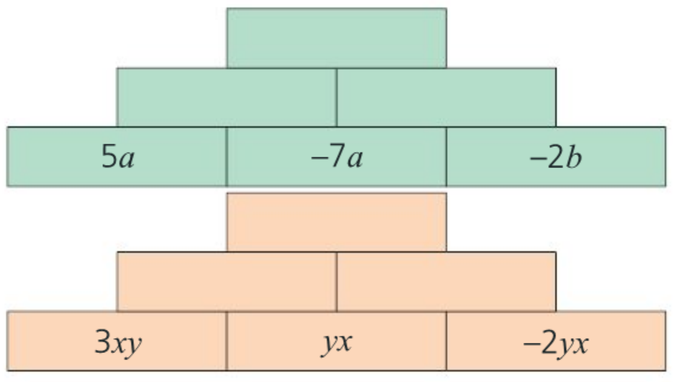


Step 3: Get the final answer at the top of the pyramid.



Try these pyramids:

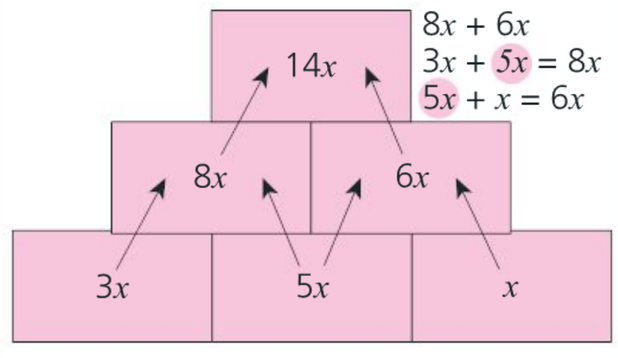




Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding and Criterion C: Communicating.

EXTENSION



How could doubling the middle box have helped you to get to the top faster?

Remember that some terms in equations are equivalent. These terms are equivalent, no matter which way round they are.

★ + ● = ● + ★

π × 3 = 3 × π

Hint

Remember simplest is best in algebra notation.

There are several agreed ways to express algebraic terms that will shorten your presentation and make it look more 'elegant':

1 If you have several variables in your final expression, list them **alphabetically**.

2 If you have several variables in a single term, list them alphabetically too: *xyz* is considered better communication than *yzx*, although they are technically the same (or equivalent).

3 List any constants (numbers only) at the end of the expression: $3d + 4$ is better than $4 + 3d$.

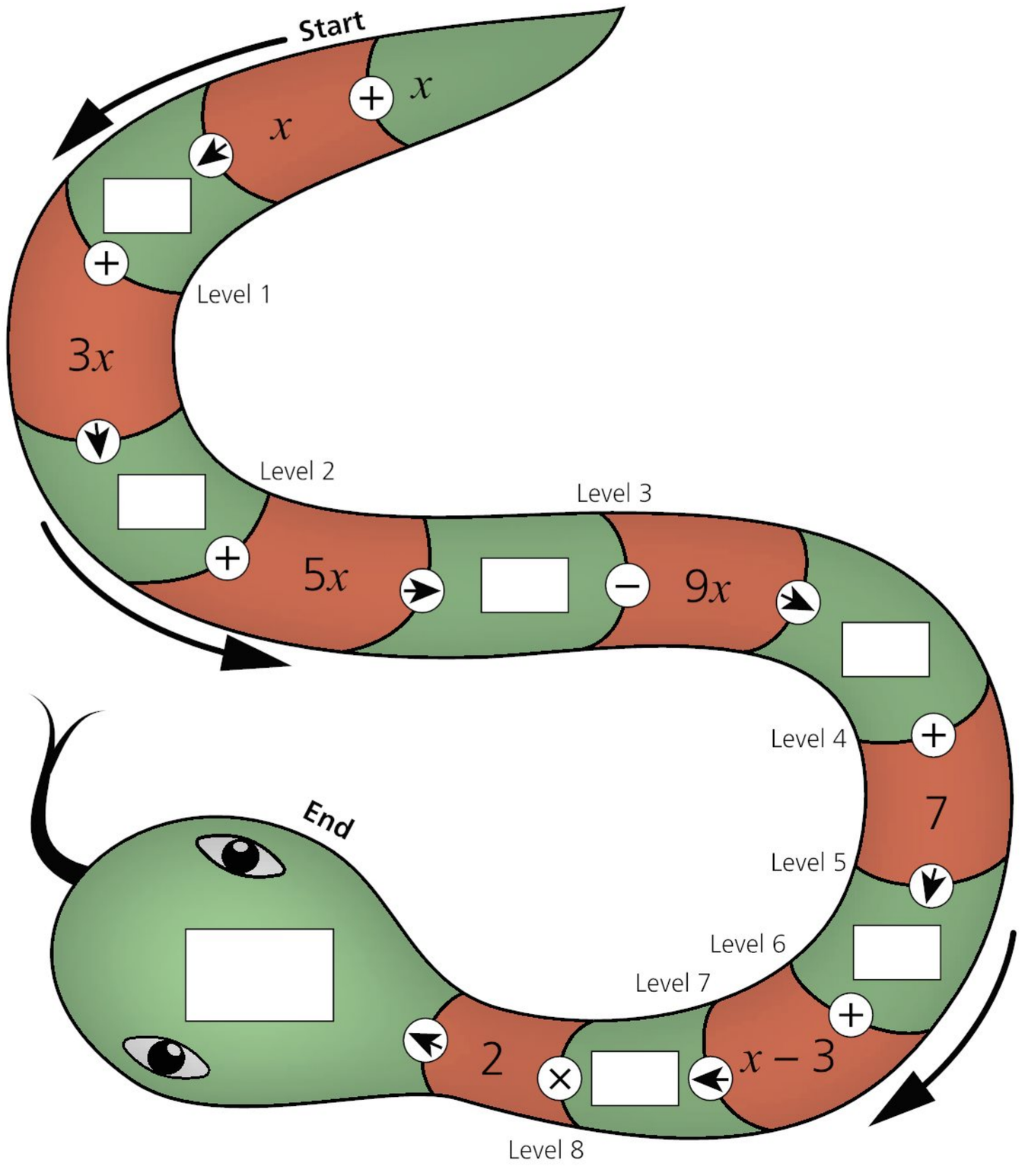
In the last example there was a constant term (4) and a variable term ($3d$). This also happens in real-life cases, such as an electrician who charges a rate per hour and a fixed (constant) call-out fee to cover travel expenses! If you call them to repair something, they might quote you two different numbers, perhaps ¥50 an hour and a ¥75 call-out charge. You could use algebra to calculate the amount you will need to pay, if you know how many hours they will work.

ACTIVITY: The snake is the ladder

ATL

- Affective skills: Demonstrate persistence and perseverance

Play snakes and ladders with a difference! Carry out each operation to give you the value for the next box. Continue all the way along the snake.

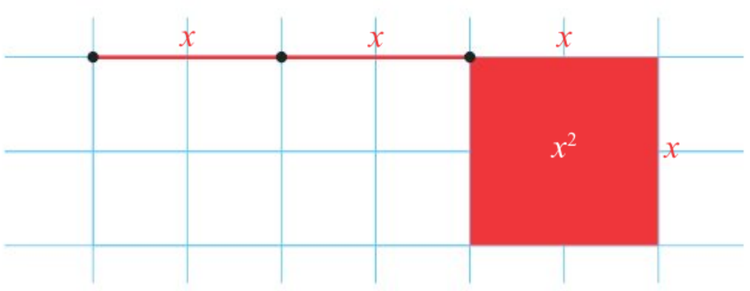


What are ‘unlike’ terms and what do I do with them?

ARE AND ^{2} LIKE TERMS?

They do contain the same *letter* but they are not the same! Sometimes people get confused and try to add x and x^2. However, these terms are definitely **not** like terms. Like terms have to be multiples of the same symbol only.

We can demonstrate this diagrammatically. Look, x and x^2 are definitely not the same.



How does this diagram explain why we can’t add x terms to x^2 terms? Give two differences between x and x^2 that tell you they are not the same and therefore can’t be added.

Integers are all whole numbers, both positive and negative.

Terms can have negative coefficients. In other words, the number in front of the symbol might be preceded by a negative sign (–). We can still treat these as multiples, even though they are negative versions.

$4 \quad -9 \quad = -5$

If we handle the coefficients only, then

$4 - 9 = \ -5$

So,

$4 \quad -9 \quad = -5$

Here’s a more complicated example:

$2 \quad +6 \quad -11$

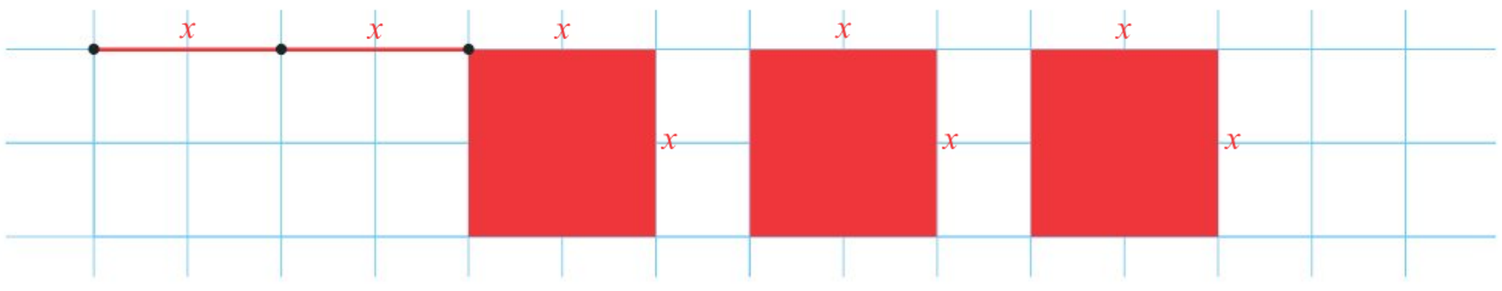
Therefore, $2 + 6 - 11 = -3$

$2 \quad +6 \quad -11 \quad = -3$

How do you solve or combine ‘directed numbers’ or integers in your mind? Do you picture a number line whereby you move to the right to add and move to the left to subtract? Do you imagine a thermometer scale and move up for positive and down for negative? Do you draw or imagine positive and negative ‘blobs’ in battle? Or do you have another numerical method, such as subtracting the smaller number from the bigger and then always use the sign from the larger one? Look back at page 6 if you need to refresh your memory on the different methods.

PRACTICE EXERCISE

Can you translate this diagram into an algebraic expression?



Explain your method to your partner and demonstrate your method in action. In return, you should listen carefully to your partner’s method. Is their method better or easier than yours? You should always remain open-minded to others so you can learn from them. If you still believe your method is better, try and convince your partner of that fact by making a persuasive argument.

WHY CAN'T YOU ADD LETTERS AND NUMBERS INTO A SINGLE TERM?

Numbers on their own don't change. If they are in an expression, we call them **constants**. The value of numbers doesn't vary, so we can't simplify them or 'conjoin' them with variables. Here's an example:

$$x + 3 + x + 4 = 2x + 7$$

PRACTICE EXERCISE

Simplify:

1. $a + 3 + a + 2 =$
2. $3b + 4 + 2b + 10 =$
3. $c + 5 + 2c + 1 =$
4. $d + e + 4 + d + e =$
5. $4f + 3 - 2f + 2 =$
6. $9g - 2 + g + 11 - 10g =$
7. $5h + 5i + 10hi + 11i =$
8. $4.5j + 4.5 + 2j - 2.3 =$
9. $-k + 100 + 42 + 5k - 230 =$

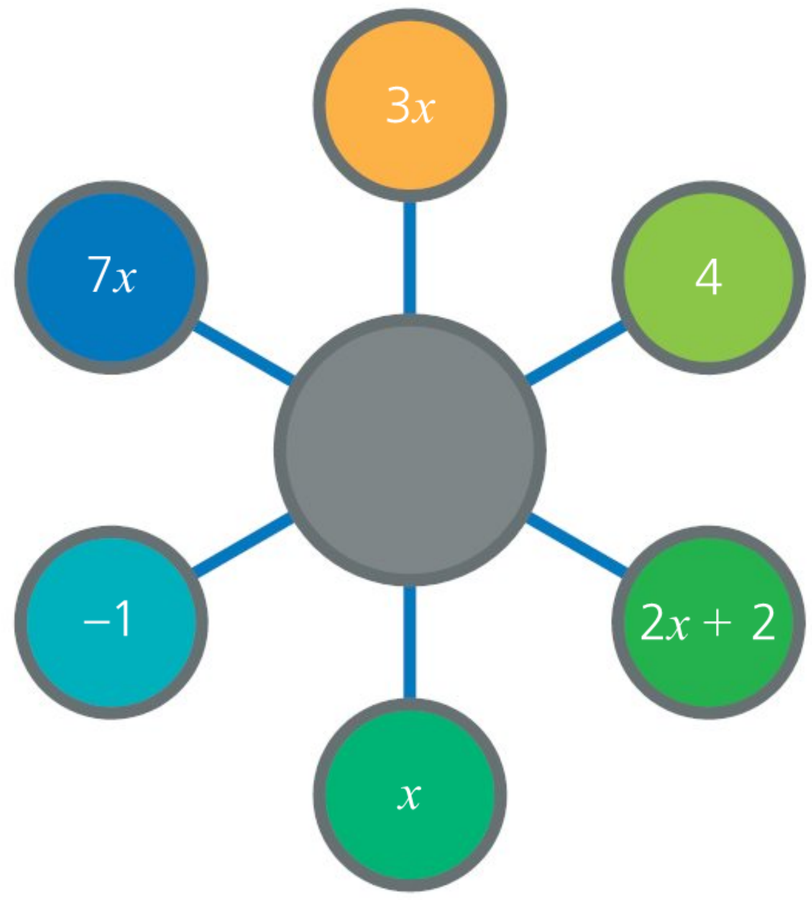
PRACTICE EXERCISE

State whether or not it is possible to add these terms to make a single term:

1. $\frac{1}{2}q$ to $0.5g$
2. $-2k$ to $+ 2k$
3. $\frac{1}{3}d$ to $0.75d$
4. 50^2 to x^2

PRACTICE EXERCISE

Simplify the terms in the diagram by addition (adding every circle). What is the answer?



Where did you start? How did you work it out?

ACTIVITY: Is it true?

Eanna makes the following claim.

If

$$a + b = b + a$$

then it must be true that

$$a - b = b - a$$

Investigate whether Eanna's claim is true or false. Communicate your thinking clearly and demonstrate by using an example if you can.

EXTENSION

Certain operations are said to be **commutative**. Research what this means. How can you connect the definition to Eanna's claim?

What are algebraic products?

Remember, in algebra **simpler is better**. Also, **communication is important** and good communication leads to mutual understanding.

SIMPLIFYING ALGEBRAIC EXPRESSIONS BY MULTIPLICATION AND DIVISION

Remember ab is the same as ba.

Therefore $ab - ba = 0$

just as $ab - ab = 0$

Multiplying different letters, symbols and numbers

- When you write a product, the coefficient comes first. This means putting any number at the front of the term.
- Use alphabetical order within the terms and between terms, if possible.
- Leave out the multiplication symbols: remember three lots of five or three times five is often simplified to 'three fives'.

Example

1 Write the product of y, $2x$ and 4 as an expression.

Solution

$(y)(2x)(4)$	$8yx$	$8xy$
OK	Better	Best

2 Improve the communication of these terms:

a $h(g)(8) =$

b $6tg =$

c $7 + yx =$

d $bac + 2abc + 8cba + 2 + 9cab =$

ACTIVITY: Communication is important

ATL

- Affective skills: Emotional management

Think about times when communication plays an important role in relationships. Write a silly algebraic equation to demonstrate this fact. For example:

$u + a = f$

where

u = how upset someone is a = apology

f = forgiveness for whatever upset them

or

$w + w \neq r$

which stands for two wrongs don't make a right!

Write your own silly equations with variables and constants but describe things about your own relationships using the letters and numbers in the equations.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

SIMPLIFYING WITH POWERS

As you have already learned, a **power**, **index** or **exponent** is used to indicate when a number is multiplied by itself. So $a \times a$ should be written as a^2.

Which means

$a \times a \times a = a^3$

$a \times a \times a \times a = a^4$

And so on ...

Links to: your own relationships with others

What could we take from the activity on page 44? How could the rules of algebra work in real life? Is communication important in relationships? Is there a right way or a wrong way to be with others? Is simpler better?

This idea really helps you to simplify if you are looking at terms like these:

- $aaaaaaaaaaaaaaaa$ which can be simplified as a^{16}
- $xxxxyyxxyyyx$ which can be written as x^7y^5
- $a(b)(a) \times a(b)(b)$ which is communicated better as a^3b^3.

In multiplication, since initial order is unimportant, communication should be alphabetical.

$2x + 5x + x^3$

like unlike

PRACTICE EXERCISE

Simplify:

1. $f \times f \times f \times f \times f \times f =$
2. $g \times g \times g \times g \times g \times g \times g =$
3. $(u \times u \times u) \times (u \times u \times u \times u \times u) =$
4. $hhhhhhhhhhhhhhhhhhhhhhhhhhhhhh =$
5. $w^5 \times w \times w \times w^2 =$
6. $ppppppppp \times ppppppppp$
7. $(ppppppppp \times ppppppppp)^2$

Hint

Striking through the terms when calculating the exponent or power helps you to keep track, so you don't forget any of the terms.

DISCUSS

You learned a long time ago that 4 + 2 is the same as 2 + 4, and this is true for all addition operations. This operation is said to be **commutative**, as we saw in Eanna's example earlier (page 43). It isn't true for subtracting, however, as 4 – 2 is definitely not equal to 2 – 4!

Later you also learned that 4 × 2 is equal to 2 × 4. This is also true for all multiplication, an operation which is also **commutative**. Division isn't **commutative**, as $\frac{2}{5}$ isn't equal to $\frac{5}{2}$.

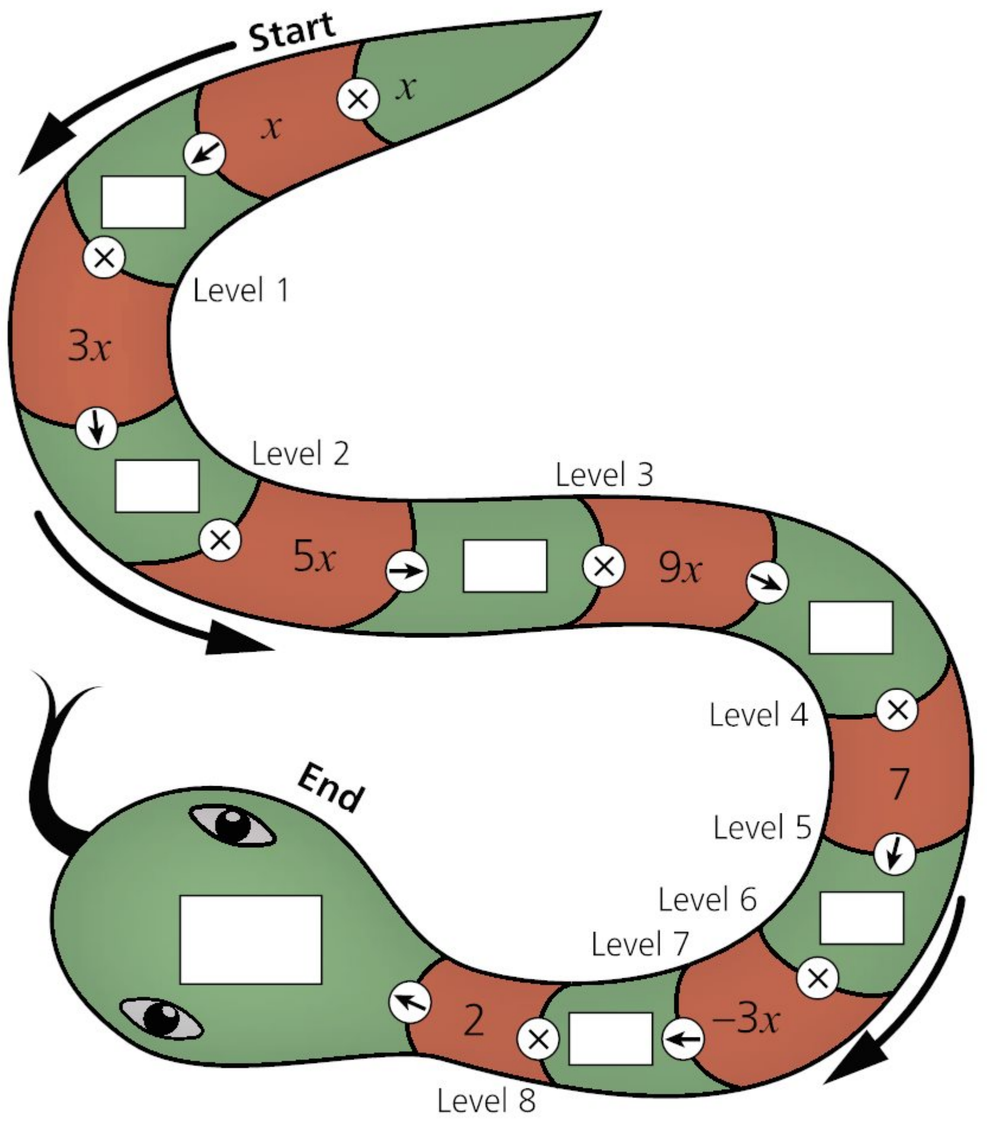
How could you express these ideas very simply for all numbers? Communicate clearly using algebraic symbols and ≠ or =.

ACTIVITY: The snake is the ladder II

ATL

- Affective skills: Demonstrate persistence and perseverance

Now you know more about algebra, try a more difficult version of the snake game.
This time we are multiplying.



Remember algebraic symbols with **different powers are not like terms**.

What happens if we have a negative term? Or more than one term?



Do you need to revise multiplying and dividing with negative numbers? Look back at pages 10 and 11.

PRACTICE EXERCISE

1 Simplify $x(x)(x)$.

2 Simplify:

a $r^5 \times t^3$

b $de \times de$

c $de \times de \times de$

d $abc \times def \times ace$

3 Go backwards! List some possible questions that could have led to these answers. Use questions 1 and 2 as inspiration.

a x^2y^2

b $\frac{x^3}{y^2}$

ACTIVITY: Random colour relationships

ATL

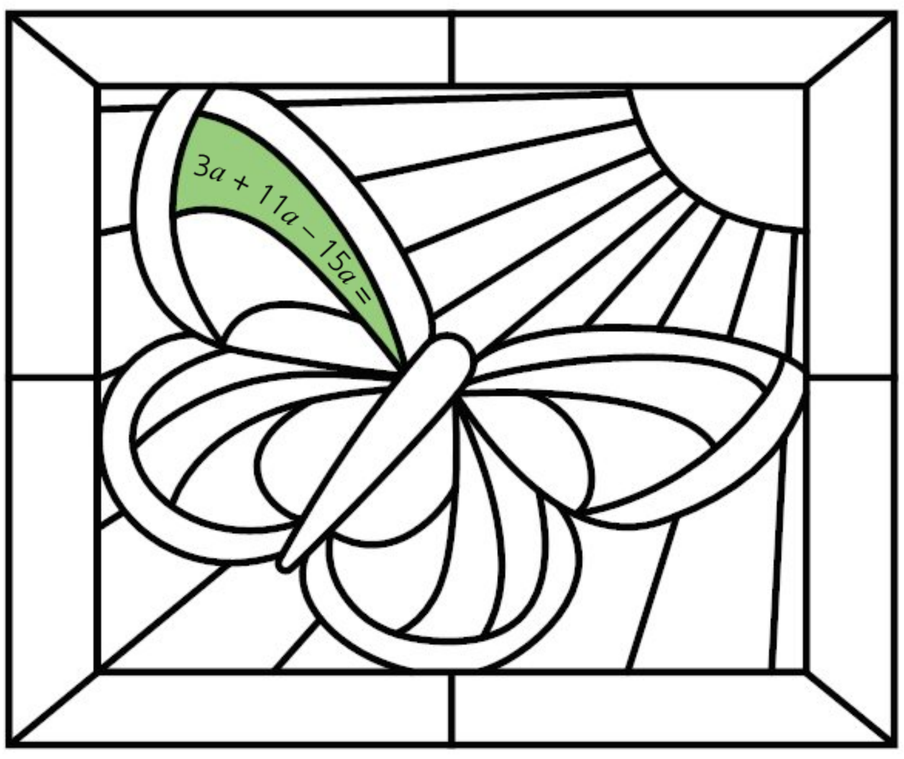
- Creative-thinking skills: Create original works and ideas

In this activity you are going to turn the window in the diagram into a stained-glass window. To colour in each section of the window, you need to think of an algebraic expression and then decide which colour to use according to the key given below. You will see one has already been put in for you.

Draw your own version of the window on a piece of paper. In each blank space make up your own algebra problem. Keep each problem with one like term, if possible, so you get a single-term answer.

Make sure to include some easy and some hard questions (use ones which you could solve yourself). Now simplify them. Look carefully at the answer, especially the coefficient (or number in front of the term). Then choose the correct colour for that section, using the list below and colour it in.

- **Positive even coefficient – yellow**
- **Positive odd coefficient – red**
- **Negative odd coefficient – green**
- **Negative even coefficient – blue**
- **Decimal or fraction in the coefficient – pink**
- **Coefficient greater than 100 – brown**



Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

WRITING ALGEBRAIC EXPRESSIONS

When thinking about writing algebraic expressions, it might be useful for you to revise the following terms. What does each one mean? Can you write an example for each?

- sum
- difference
- product
- quotient

Wow! You are on your way to becoming fluent in algebra. You can now translate whole phrases in and out of this exciting mathematical language. Next stop, sentences (see Chapter 4)!

PRACTICE EXERCISE

1 Translate the following expressions from algebra to English by matching an expression from the left-hand column to a description in the right-hand column.

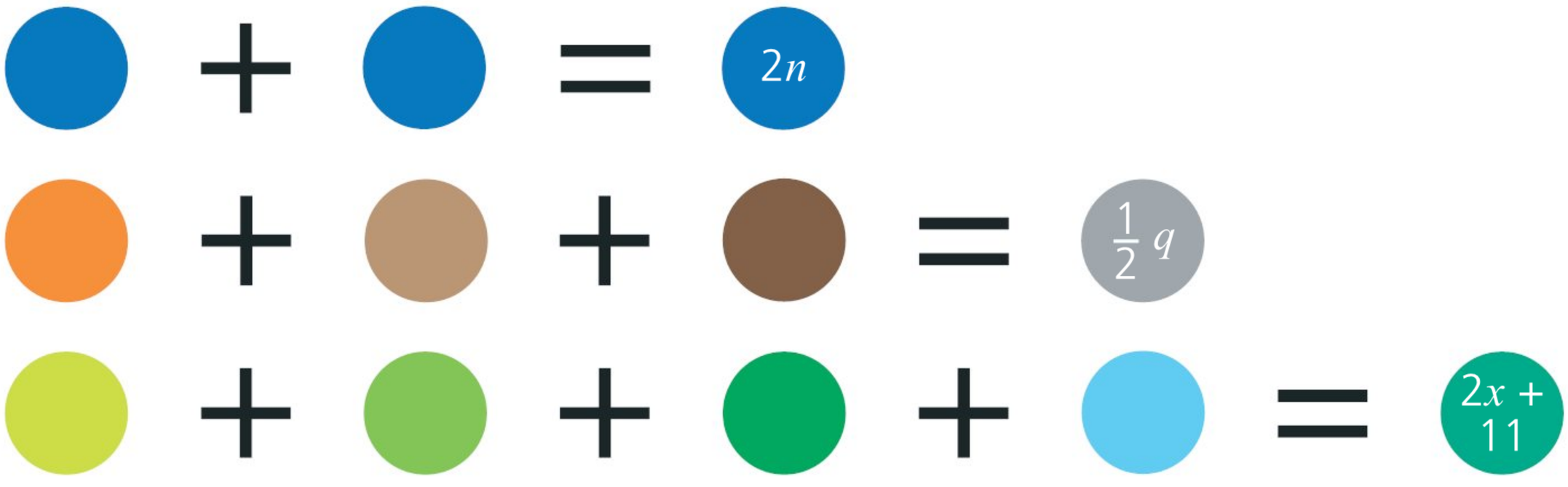
$3n$	A number added to itself
$2n$	n added to another n and to another n
$n + 7$	A number multiplied by itself
n^2	A number subtracted from 8
$8 - n$	A number added to 7
$n - 8$	A number minus 8

2 Write down:

- **a the sum of b and c**
- **b the sum of b and c squared**
- **c half of a plus b**
- **d the sum of half a plus a times b**
- **e twice the product of x and y**
- **f the product of d and c**
- **g the product of l and g squared.**

ACTIVITY: If this is the answer, what was the question?

In these equations the answer has been given, but not the expressions. Work backwards from the answer to **suggest** what the expressions could be.



Are variables the key to understanding relationships?

ACTIVITY: Algebra tiles – spotting patterns and simplifying them

ATL

- Communication skills: Organize and depict information logically

This investigation is a guided one, with lots of steps to help you find the right information and, hopefully, guiding you to see patterns. Your job is to find the patterns and describe them. Due to the guided nature of the investigation, 6 is the maximum level you can get. When you become more familiar with Criterion B investigations, they will become more **open-ended**.

You can also have additional teacher support if you are stuck, but remember this can affect your achievement level, bringing it down. Use it only if you really need it, not just to check if you are on the right track.

Part A



Shape 1: Imagine a shape like this, where the long sides are twice as long as the short sides. We don't know the exact length of either the long or the short sides, but we will call the short sides x.

What would the length of the long sides be?

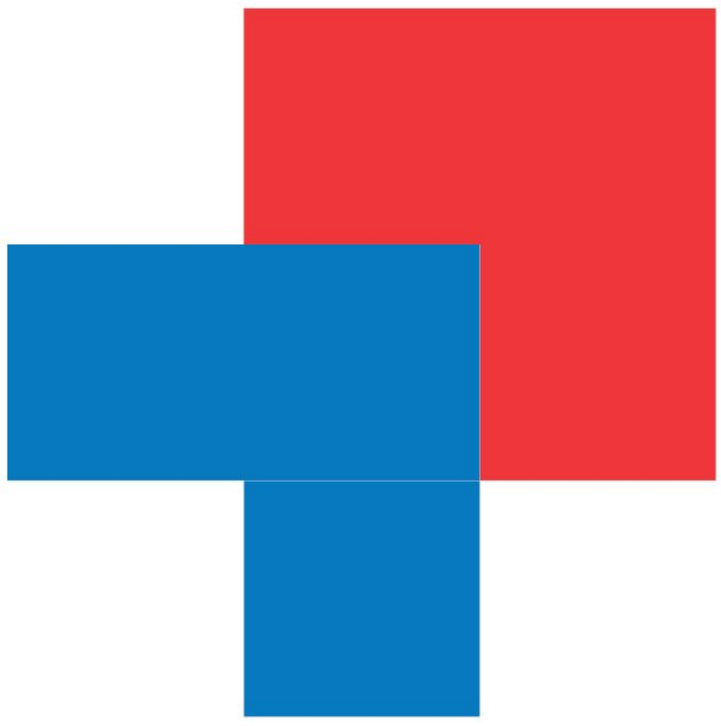
Copy the shape and **label** the sides using x.

The **total** outside length of a shape is called the **perimeter**. Find the perimeter of this shape by adding up all the sides to find the total length.

Hint

Stuck? Can't get started? Look at page 49 for a hint on labels for the sides. Remember, this counts as **additional support** so use it wisely!

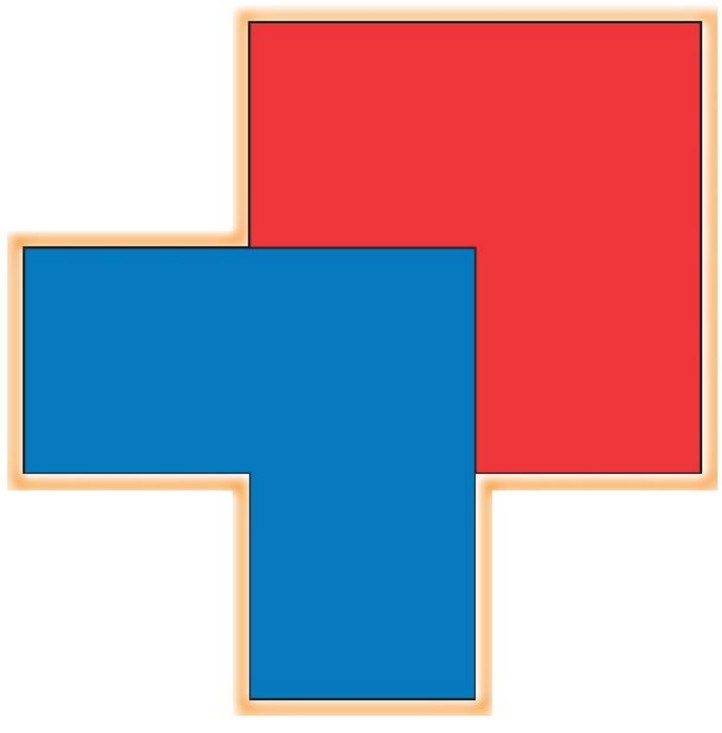
Shape 2: Now imagine that a copy of the shape is placed touching the first shape, like this:



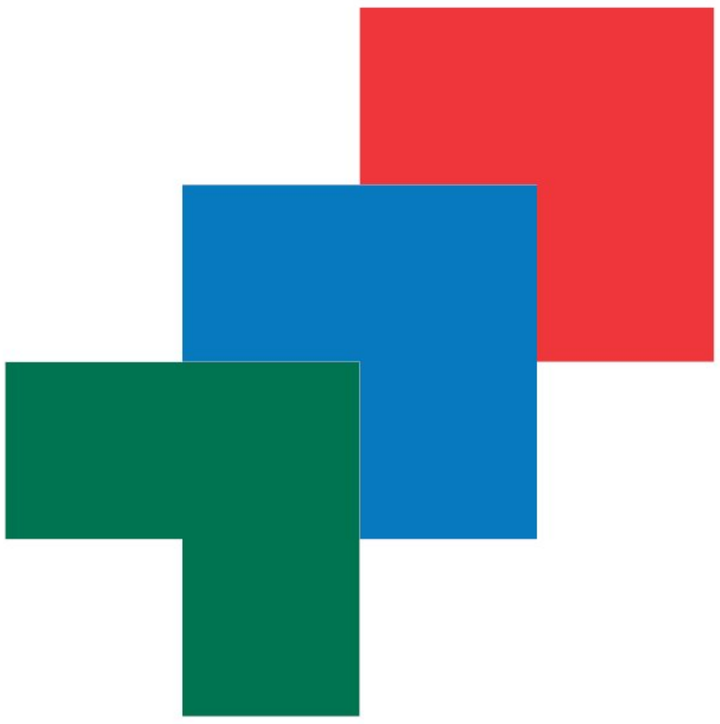
Again, **label** all the sides of the new shape.

If the perimeter is only the sides on the **outside** edges of the shape – what is the new perimeter of this shape?

If you are unsure of **where** the new perimeter is, you can use this diagram to help you:



Shape 3: By placing another copy of the original shape in the same way, we are now making an interlocking or herringbone pattern.



Label the lengths of each side of the new shape.

Find the total perimeter for shape 3.

Part B

Using the diagrams you have created, fill out the information in a copy of the table below.

Shape number	Perimeter of this shape	How many pieces were there?
1		
2		
3		
4		

Do you see any pattern, or patterns, in your answers? **Describe** how these patterns work.

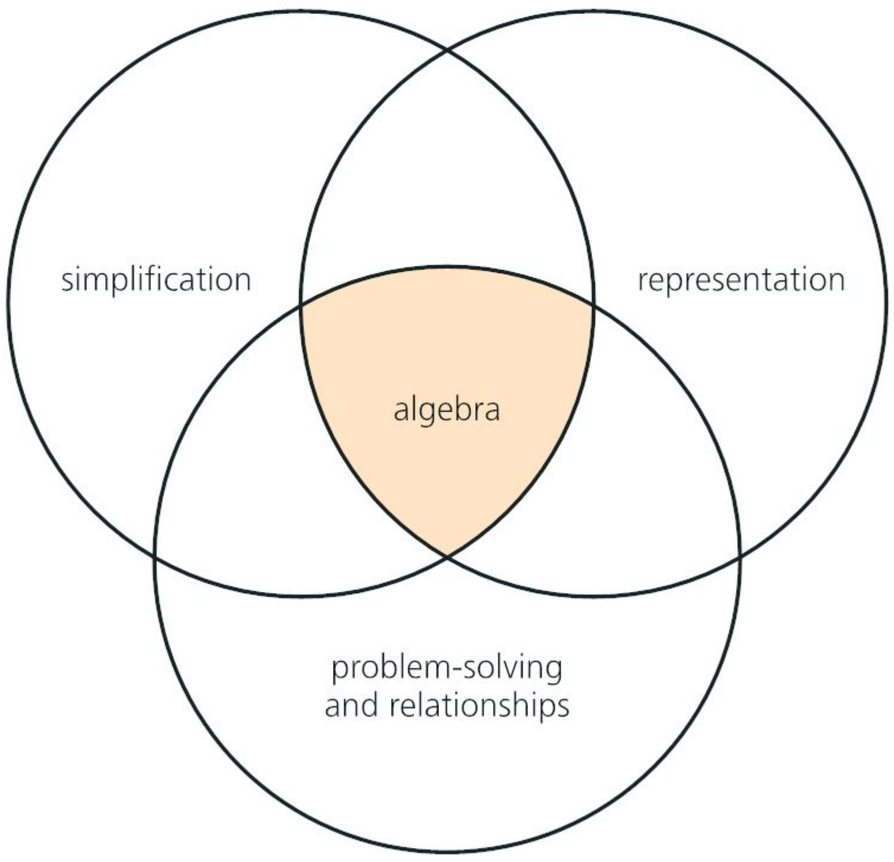
Can you find a general rule to describe the pattern, no matter how big the shape?

Predict how your pattern would work for shape 4.

Verify (check) if your prediction is correct by drawing the shape and finding the perimeter.

Assessment opportunities

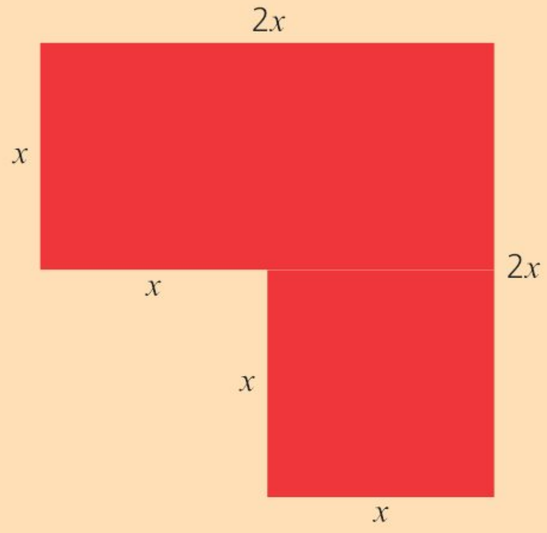
- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.



Explain what is meant by this Venn diagram. What does it have to do with your work in this chapter, as well as with the activity you have just completed?

Hint

Do you need to take the hint mentioned on page 48? Are you sure? It will mean you can't get the very highest achievement level (but it also means you won't get a zero!)



ACTIVITY: Who invented algebra?

ATL

- Critical-thinking skills: Consider ideas from multiple perspectives

The question 'who invented algebra?' was posted in early 2013 on an answers forum called quora.com: **www.quora.com/Who-invented-algebra**.

People can submit answers and other readers can 'up vote' or 'down vote' them, depending on whether they find the answers useful or correct.

On the right, you can see a post from Professor David Joyce.

A student, Dimitra Varoufakis, reads Professor Joyce's post and disagrees. She decides to post on the discussion herself and argues that because the Greek mathematician Diophantus wrote a series of books on the subject known as Arithmetica, the invention of algebra is Greek. This took place around 200 CE. She gets over 3000 views and 14 upvotes.

Another student, Somreeta Patel, thinks that algebra is even older than Diophantus and could be credited to al-Khwarizmi. However, she also knows there is more to algebra than the use of symbols, so the great Indian mathematicians Brahmagupta and Bhaskara, who developed procedures for solving equations without using symbols, could be considered the inventors. She wants to post this answer too. What would you recommend?

Read again and carefully consider David's, Dimitra's and Somdatta's arguments. Now it's time to decide … Who do you think invented algebra? Why?

Make a convincing argument to say who invented algebra. You will need to do additional research and reflection on the topic before you make your mind up. Use these search terms: **history of algebra**, **who invented algebra**.

Do you think someone who is Indian or Chinese or Arabic might answer differently from someone who is not?

How might your nationality or identity affect your decision? Do you think pride (how proud or nationalistic you are) might play a part?

Can our identity affect our mathematical understanding or beliefs? How accurate would our decisions be then?



David Joyce
Professor of Mathematics and Computer Science

Written Mar 13, 2014

It all depends what you mean by algebra.

Solutions to linear and quadratic problems

If by algebra you mean solving for unknowns when you know something about them, such as the thing squared plus three times the thing equals 144, that's ancient. The Babylonians knew how to solve linear and quadratic problems 4000 years ago. The Egyptians could solve many of those about the same time. Euclid encoded their solutions in the *Elements* 2300 years ago. The Chinese could solve them 2000 years ago or earlier. They could also solve systems of linear equations back then, too. So could the Indians then or earlier.

Methods to solve linear and quadratic problems were known in all the advanced ancient civilizations.

The word algebra comes from a word used by Muhammad ibn Mūsā al-Khwārizmi (ca. 750–ca. 850) in a book he wrote that systematically described the solutions of linear and quadratic problems by reducing them to simpler problems.

Symbolic algebra

If by algebra you mean symbolic algebra that has signs for the arithmetic operations and for equality, and symbols for variables, that was created in the 1500s and continued to be standardized in the 1600s. No one person invented that. You can point to individuals who contributed to the development of symbolic algebra, and a couple who were influential in standardizing and spreading its use, but you can't say it was the invention of any one person.

6.8k Views • View Upvotes

Upvote 7 Downvote Comment

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

MEET A MATHEMATICIAN OF YOUR CHOICE

Learner profile: Inquirer, Communicator



Our identities are complicated and include many different aspects of our being, such as our nationality, language, religion, gender, past and many other factors.

Find a mathematics discovery or invention made by a man or woman who shares an aspect of identity with you, such as nationality, belief system or socio-economic background. Create a 'profile' or poster to explain what they discovered or contributed to mathematics. Give background information on their life and works as well.

Explain how it makes you feel to have something in common with this mathematician. Make sure to include something which illustrates the relationship between you and the mathematician, such as flags, symbols or pictures.

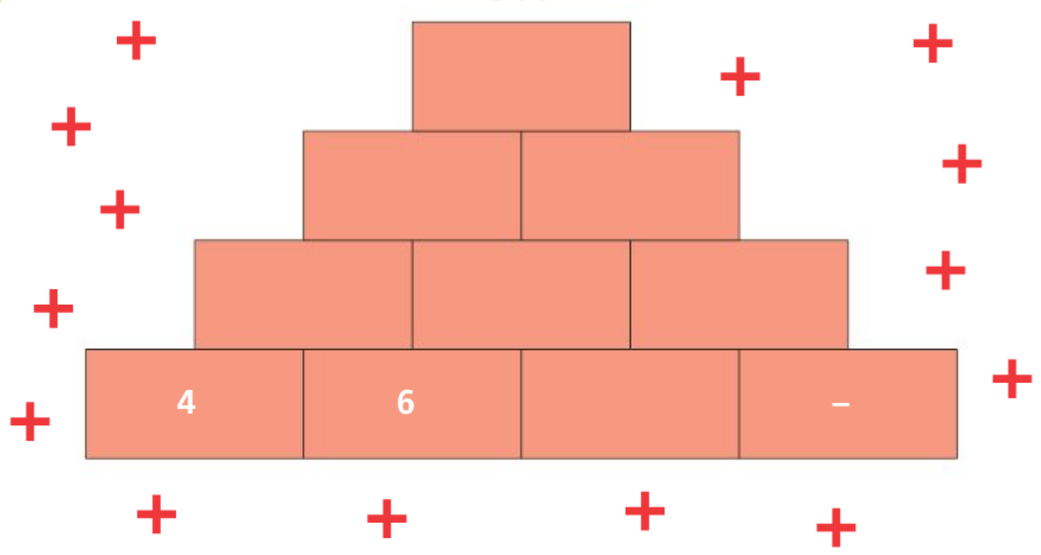
SOME SUMMATIVE PROBLEMS TO TRY

Use these problems to **apply** and extend your learning in this chapter. The problems are designed so that you can evaluate your learning at different levels of achievement in Criterion A: Knowing and understanding.

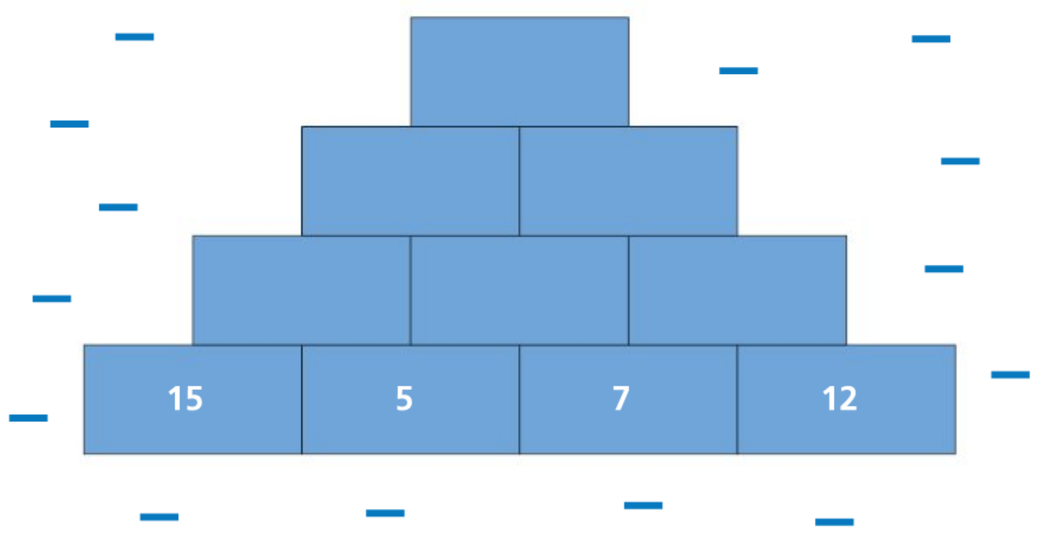
THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 1–2

1

Adding pyramid



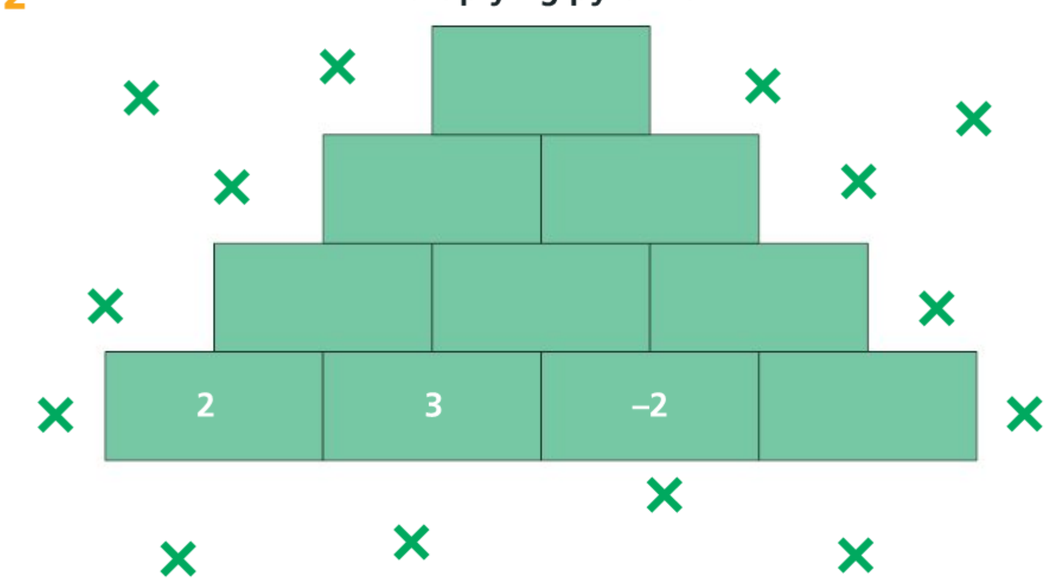
Subtracting pyramid



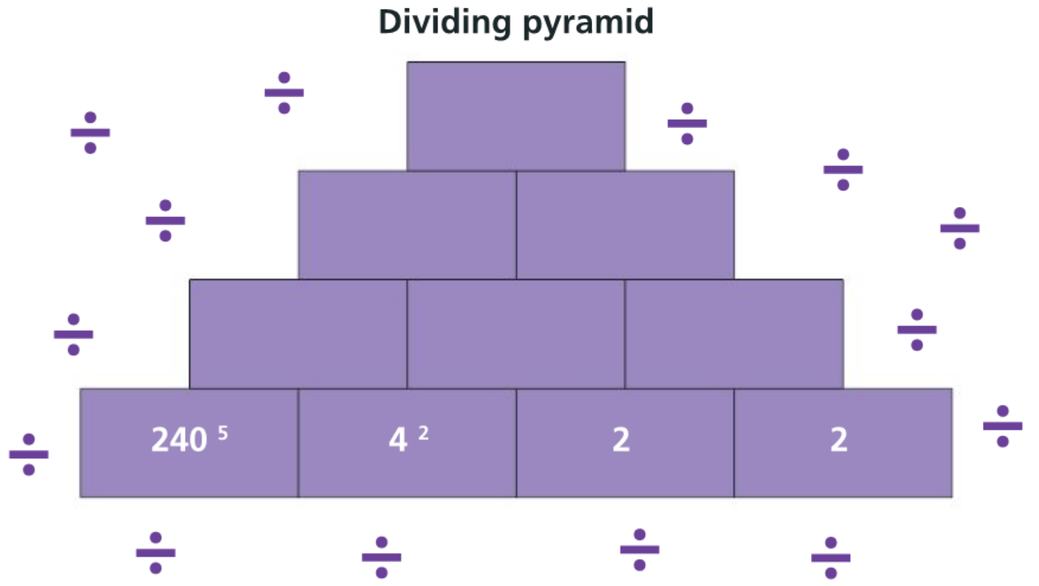
THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 3-4

2

Multiplying pyramid

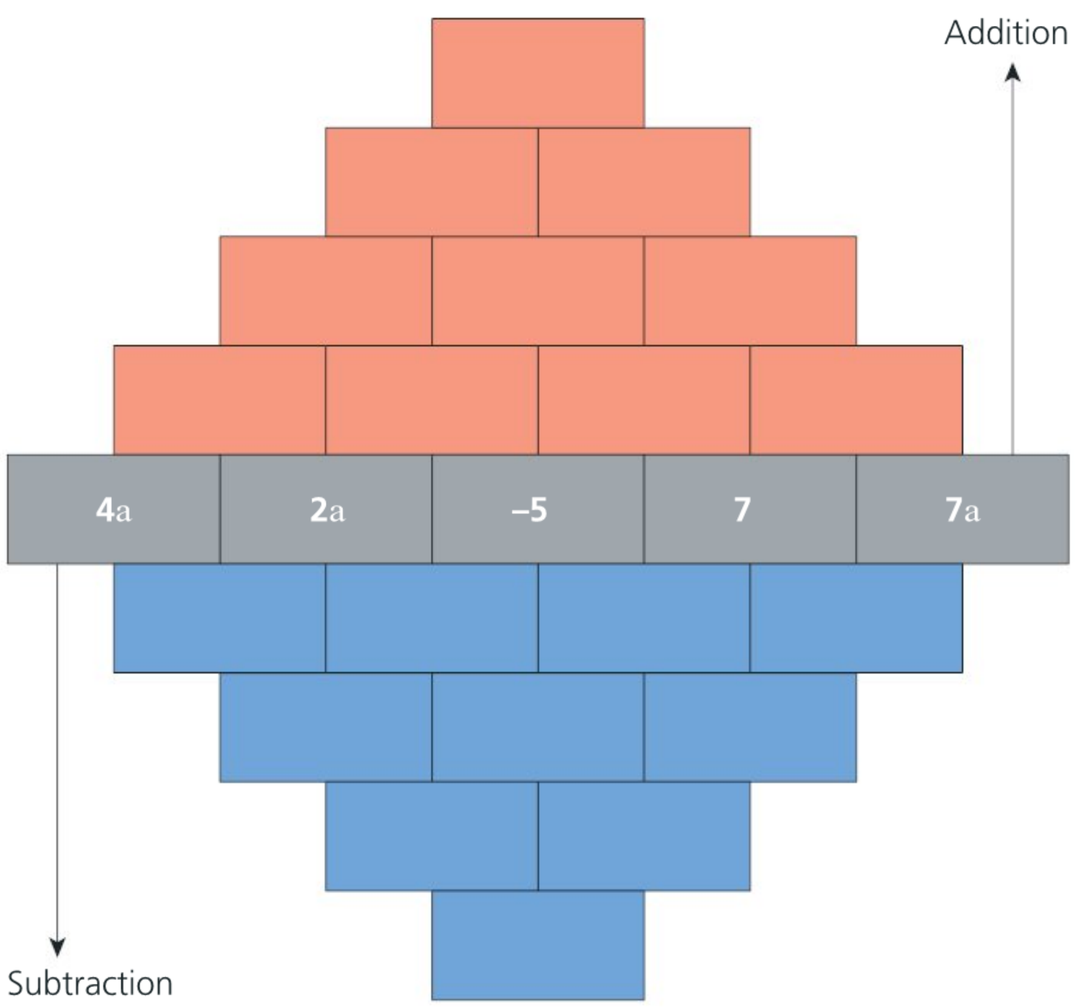


Dividing pyramid



THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 5–6

3 Complete the pyramid by adding boxes upwards but subtracting downwards.



THESE PROBLEMS CAN BE USED TO EVALUATE YOUR LEARNING IN CRITERION A TO LEVEL 7–8

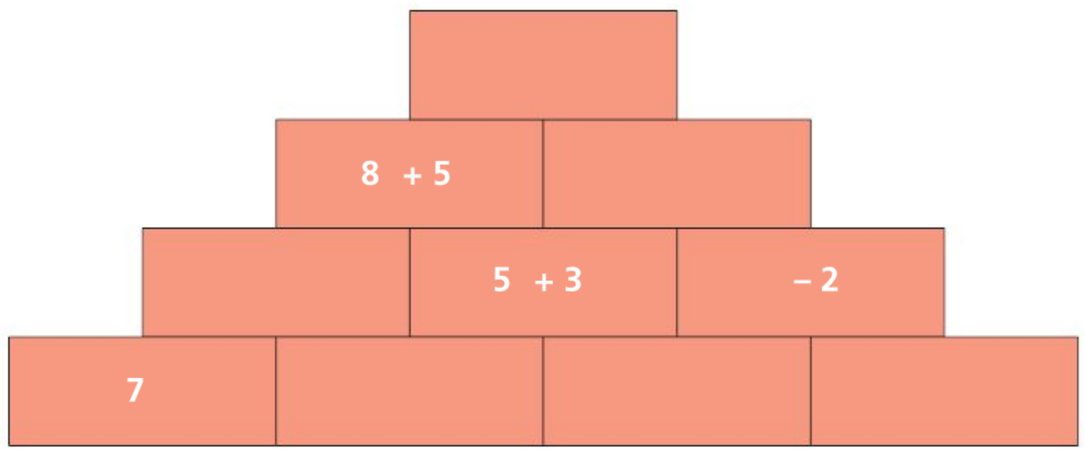
4 What is the difference between the top and the bottom single boxes in the diamond above?

5 What is the sum of the top and bottom answers in the diamond above?

6 What is the product of the answers in the diamond? You may leave your answer in bracket form.

7 What is the quotient? You may leave your answer in bracket form.

8 Complete all the blank boxes in a copy of this addition pyramid.



Reflection

Use this table to reflect on your own learning in this chapter.						
Questions we asked	Answers we found	Any further questions now?				
Factual: What is algebra? What are like terms and how do we collect them? How can I tell if terms are like or not? What are algebraic products?						
Conceptual: What is a relationship? How do we express ourselves in algebra? What are unlike terms and what do I do with them? What happens if we have a negative term, or more than one term? Are variables the key to understanding relationships?						
Debatable: Why do we need to have common rules of communication? Does it help to be the same or is it better to be different? Could algebra be a universal language?						
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?				
		Novice	Learner	Practitioner	Expert	
Transfer skills						
Affective skills						
Information literacy skills						
Communication skills						
Learner profile attribute(s)	How did you demonstrate your skills as an inquirer and a communicator in this chapter?					
Inquirer						
Communicator						

3 How can data help us save the world?



Representing data visually helps to identify **relationships** that can **justify** **global** decisions.

CONSIDER THESE QUESTIONS:

Factual: Where does data come from? How are statistical variables different from algebraic variables?

Conceptual: Who needs data? Are data just a list of numbers? What sorts of stories can the data tell us?

Debatable: Can everyone understand data? Can data provide answers to global problems? What kind of difference can one person make? Can graphs lie?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- drawing and reading graphs
- finding the mean, median, mode of data.

IN THIS CHAPTER, WE WILL …

- **Find out**:
 - how to collect primary and secondary data from good sources
 - how to represent data effectively
 - how to read, interpret and analyze data from various graphs.
- **Explore**:
 - the influence data can have on decisions in all kinds of industries
 - how new data can affect measures of central tendency.
- **Take action** by investigating an online statistical study and discussing its implications for ourselves and our peers.

We will reflect on these learner profile attributes …

- **Knowledgeable** – We develop and use conceptual understanding, exploring knowledge across a range of disciplines.
- **Reflective** – We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.



These Approaches to Learning (ATL) skills will be useful …

- Communication skills
- Information literacy skills
- Reflection skills
- Critical-thinking skills
- Transfer skills

Assessment opportunities in this chapter:

- **Criterion A**: Knowing and understanding
- **Criterion C**: Communicating
- **Criterion D**: Applying mathematics in real-life contexts

KEY WORDS

conclusion	population	sample
data	primary	secondary
frequency	relationship	trend
pattern	representation	

STATS AND FACTS

'Statistical facts don't come to people naturally. Quite the opposite. Most people understand the world by generalizing personal experiences which are very biased (prejudiced, one-sided). In the media the "news-worthy" events exaggerate the unusual and put the focus on swift changes. Slow and steady changes in major trends don't get much attention. Unintentionally, people end-up carrying around a sack of outdated facts (from outdated textbooks).'

Hans Rosling, Swedish mathematician and statistician

THINK-PUZZLE-EXPLORE

1. **What do you think you know about data?**
2. **What sort of data do you come across in your school life?**
3. **What questions do you have that data could answer?**
4. **How can you explore those questions?**

Who needs data?

In December of 2015, the Paris Climate Accord was signed by 188 countries, each committing to decrease their emissions to an agreed level. This is one of many global efforts to control one of the most concerning issues in science and politics – climate change.

How do we know that climate change is a threat? How are scientists able to show that climate change is happening at a rapid pace? In 2015, how were they able to convince the world's politicians to do something about it? The answer to all these questions is the same – **data**.

By recording trends in weather patterns, solar radiation levels, glacier thicknesses, ocean depths and other similar observations over time, scientists have been able to draw conclusions about global warming and to convince others of its gravity (seriousness). Simply warning country leaders about a potential disaster won't inspire them to take action, but backing up these claims with data – concrete evidence – does affect decisions. Data are essential to making informed and sound decisions.

When using data in decision making, there is an entire process to go through before the numbers can tell you their story, as the flow chart below shows.

ACTIVITY: Data are used in all walks of life

ATL

- Transfer skills: Inquire in different contexts to gain a different perspective

Look at the people in the pictures on page 55. Write a few sentences to **explain** how each person might use data to help make decisions, and include examples in your response.

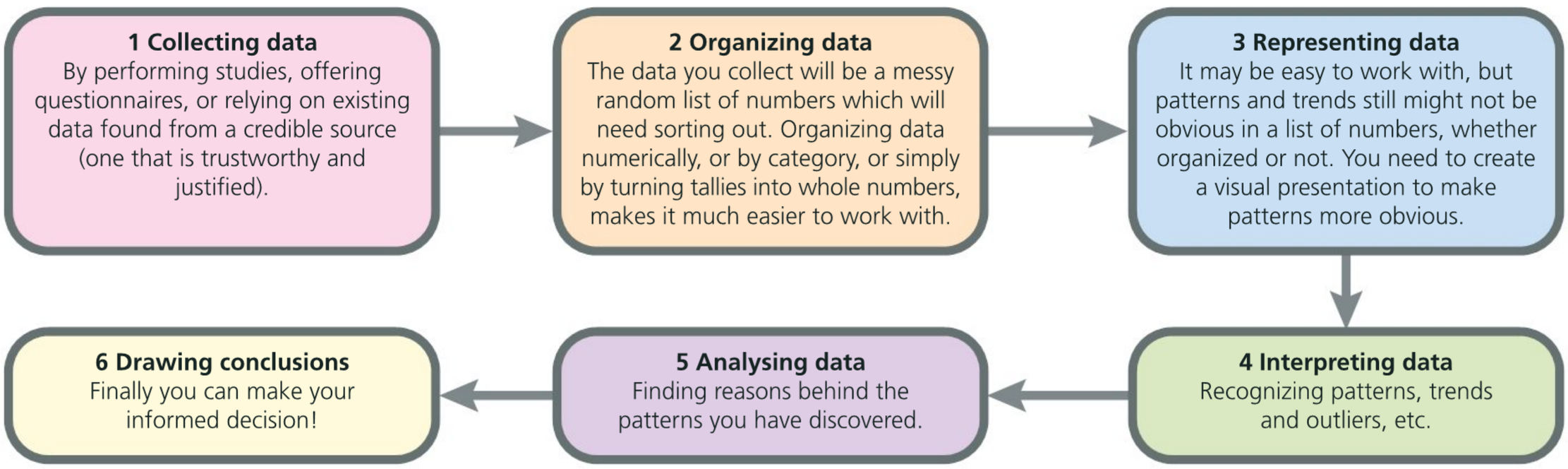
Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

Take action

! Find a data study that has been reported online or in a magazine or news desk of your choice, and identify the six steps shown in the flow chart. Are any steps missing? If so, why do you think the reporter chose to leave out missing pieces? Present your findings to the class sticking as closely to the format below as you can. Discuss how the results of your chosen study may influence you or your peers.

A statistical study follows a similar structure to that of a lab report. Opposite is an example of a statistical report about the declining African rhino population.



MEET A MATHEMATICIAN: FLORENCE NIGHTINGALE (1820–1910)

Learner profile: Knowledgeable and caring



Known as 'the lady with the lamp', Florence Nightingale was an English nurse in the 19th century. During the Crimean war, she made rounds of the wounded at night with a lamp, secretly collecting data as she cared for patients.

She would note causes of illnesses and keep a running count of those who died. Knowing that most people would have trouble reading her statistical reports, Nightingale created the first-ever graphical representation of data – **the polar area diagram**. With this graph, Nightingale was able to convince Members of Parliament and civil servants to improve medical care conditions.

Her knowledge of good nursing practices and mathematics, combined with her ability to reflect on her data, helped to create hygienic standards and other improvements which led to a decrease in deaths from 42% to just 2%!

New nurses must now take the Nightingale pledge, and International Nurse's Day is observed worldwide on her birthday.

Change in African rhino population in various years

Trappers catch and tag African rhinos, recording how many they have trapped. Then they use a formula to estimate the total population. This is more practical than observing every rhino in the country.

As this process repeats from year to year, a table of results is created.

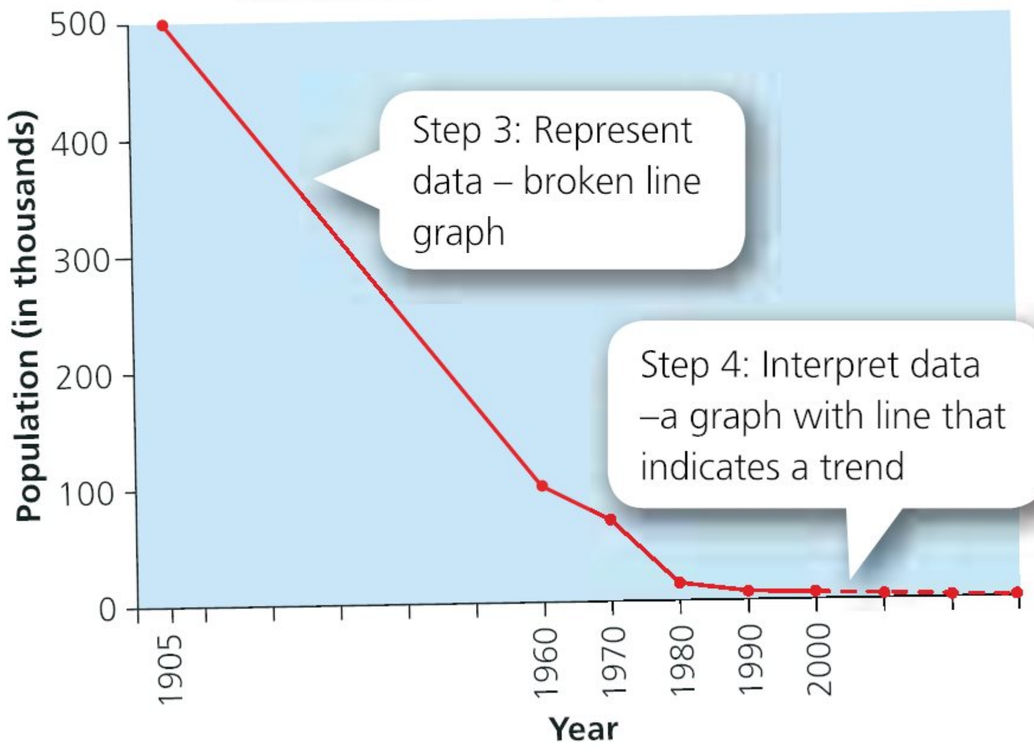
Step 1: Data collection – observing rhino

Year	Population
1905	500 000
1960	100 000
1970	70 000
1980	15 000
1990	3500
2000	2400

This table of numbers has limited use. The data need to be displayed in a way that will show a trend over time. A broken line graph is perfect for this:

Step 2: Organize data – table of raw data

African black rhino population over time



Step 3: Represent data – broken line graph

Step 4: Interpret data –a graph with line that indicates a trend

This line graph indicates a serious population decline. The curve shows a rapid decline in numbers until the 1980s. After this the population continued to decline, but not as quickly as rhinos were so rare they were harder to hunt. We can draw a dotted line that follows this trend to predict future outcomes.

Why are we losing rhinos each year?

Step 5: Analyze data – look for reasons behind trend

It turns out that early 20th-century sports hunting was just the beginning of rhino decline. In the decades that followed, rhino habitats were cleared for agricultural purposes. Then Chinese medicine hailed powdered rhino horn a cure-all product, and some cultures in the Middle East used their tusks for ceremonial knives.

If we follow the trend line, and no changes are made to protect the African black rhinos, we can predict that their extinction will take place in the year _____.

How might we use the data and analysis to help protect the African black rhino? Who would we share it with and what would be the next steps?

Step 6: Drawing conclusions – predict date of extinction; suggest ways to prevent it

The outcome

Thanks to the research performed above, conservation programmes were put in place and, since the early 1990s, the African black rhino population has, in fact, risen to 5200 by 2015.

Where does data come from?



Census in action! Google's camera car

PRIMARY DATA

Whether carrying out experiments, surveying your peers, or sitting in a shop making observations, you are the one collecting every piece of data that you will analyze. You are collecting **primary data**.

Surveys

You might want to make an analysis about a population, for example all 13-year-olds who attend school or all lung cancer patients, but you can't reach each person in that population because it is too expensive, too time-consuming, or they are just not accessible to you – or perhaps they don't want to answer your questions! In these situations, investigators create a 'sample population' or a subset of the subjects (people or even animals) they are studying and, as long as this sample is large enough, it can represent the entire population.

This is often done in voting polls to predict future leaders of countries, in testing medicines (sometimes the initial sample consists of rats instead of humans), and in many other important contexts.

Census

There are situations when it is crucial that **every single subject** in a population must be accounted for, and a sample is simply not accurate enough. This can be a very expensive and extremely difficult exercise to carry out.

Many countries carry out a **census** of their residents at regular intervals to help leaders make decisions about how to allocate tax money. For instance, if there is a spike in births, they will know that they may need to build more schools within the next five years. If there is an ageing population who may soon retire, the government may encourage young working people to move to their country, as Sweden has done. The data from national censuses are so crucial to governments that many countries issue fines to those who don't complete them. In Australia, those who don't submit their census on time face fines of up to $180 per day, and in the United States censuses containing misleading information can lead to a $1000 fine and a year in prison!

Google uses a camera car to ensure that every possible address you input has an associated image – in this case the 'population' is street addresses and the census is the act of taking pictures.

Observation

Sometimes a survey may ask its sample population a question the members can't answer accurately, like how much trash they throw out in a day. They can give you their best guess, but the only way to know for sure is to make and record observations. In this instance, investigators would have to go through the sample population's trash each day for a week, and then average the amount they have thrown out to determine their daily amount.

Links to: Sciences

Observation in sciences is often referred to as experimentation. You will have carried out experiments in which you have looked for data to show relationships between variables, and you may have had to find averages too!

There are other situations in which a survey just can't be used: determining the effect of a medication on a rat, for instance. Since we can't ask the rat how it is feeling, we would need to monitor its behaviours and vital statistics to notice any changes.

SEE-THINK-WONDER

These people are making observations for research. What might be some questions they are trying to answer?



SECONDARY DATA

Often you will need to use data that have already been collected by larger organizations, such as the police, social media sites, medical companies and health-care providers, sports websites or stock exchanges. It is easy to automate the data collection process, but it takes a human being to make sense of the data – to identify trends and to spot **outliers** (data which doesn't follow the general trends), and to make predictions. The key in using secondary data is to ensure your sources are **credible**.

In 2014 Maclaren, a Formula One team, hosted a race with 20 cars on the grid, each equipped with 150–300 sensors. During the 2 hours of the race, more than 248 TB (terabytes) of data were generated and broadcast to 10 analytic centres across the world. To put this into perspective, this is the equivalent of approximately 83 000 HD movies.

How do you think the Formula One racing data could be used?

During the race, 150 000 live simulations of that mass data set took place **in real time**. This means that computers tested 150 000 'what if' scenarios, so that engineers could make adjustments to car settings to influence the outcome of the race.

ACTIVITY: Gathering data

ATL

- Communication skills: Organize and depict information logically
- Critical-thinking skills: Consider ideas from multiple perspectives; Gather and organize relevant information to formulate an argument

How might you gather data to answer the research questions below? **Justify** your responses.

- **Should the country put more money into its pensions?**
- **Does licorice make a sore throat feel better?**
- **How much money does an average Premier League soccer player earn?**
- **Which companies have had a steady increase in profits over the past three months?**
- **How many birds live in the park nearest to me?**
- **What is the most popular television programme in my school?**
- **Can rhino horn actually cure cancer?**

Assessment opportunities

- In this activity you have practised skills that are used in Criterion C: Communicating.

DISCUSS

Is the Maclaren data primary or secondary data?

The event took place in 2014. What are some of the amounts of data being used or downloaded today? Use these search terms: **amount of data uploaded to facebook**; **number of bytes in a song file**; **amount of data used by NASA**.

The act of recording data via sensors is called **telemetry**. Does the use of telemetry for real-time simulations make F1 races more or less fair than before? Why?

How are statistical variables different from algebraic variables?

In the last chapter, we saw that a variable is a symbol that can have different values. In statistics, we use the word 'variable' to refer to the data we are measuring or studying. These variables can be different quantities, such as a plant height of 45 cm, or they can be different qualities, such as the colour blue. For more on variables, see *Sciences for the IB MYP 1: by Concept*, page 6.

QUANTITATIVE DATA

Here are some examples of **quantitative** data. Can you think of three more?

- The weight of newborn babies in Kenya
- The distance children travel to school in Indonesia
- The number of texts received in a day by Danish teenagers

QUALITATIVE DATA

Here are some examples of **qualitative** data. Can you think of three more?

- Most popular song title in the UK this week
- Eye colour
- Symptoms after taking a prescribed medication for one week

Sometimes, we focus on one single variable to study (**uni-variate data**), such as the current most popular television show among 13-year-olds. More often, however, we look at two variables (**bi-variate data**) **simultaneously** (at the same time) to see if there is a relationship. How does one affect the other, if at all? Consider the height of a plant over time. Height is one variable, and time is the other. Here, one variable depends on the other. How tall a plant grows depends on the amount of time that has passed since it was planted (assuming it has been well cared for).

DISCUSS

Are each of the studies below uni-variate (mono-variate) or bi-variate? What might the variable(s) be?

- **The effect of the climate on ice-cream sales**
- **Favourite vegetable in a school survey**
- **Top 20 songs downloaded on iTunes**
- **Whether eating more organic food increases our energy levels**
- **Whether wealth affects academic performance**

ACTIVITY: So many variables

ATL

- Communication skills: Use and interpret a range of discipline-specific terms

This is a phrase you often hear with respect to a very complex issue, like sustainability or climate change. 'There are so many variables' means there are many things (or **factors**) that can affect an issue, making it very difficult to study. Below are just a few of the many variables or factors that affect sustainability. Studying these factors can help us determine and justify any changes the human race must make in order to remain on the planet:

- **Human population**
- **Sustainability**
- **Available farmland**
- **Available water**
- **Laws**
- **Air quality**
- **Policies on consumption (using up of resources)**

Can you think of two more variables you could add to the list? Which involve quantitative data and which involve qualitative data? Which data could you find in other sources and which would you need to collect yourself? How would you begin to collect data on these variables?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

Is data just a list of numbers?

REPRESENTING QUANTITATIVE DATA

The table shows quantitative data after it has been collected. It is very difficult to notice any patterns or to draw any conclusions just by looking at this list of numbers. It is also a very boring presentation!

Once data have been collected, they need to be represented visually in a way that will help the observer to better analyze it. How can we turn a bunch of numbers into a picture that tells a story? That depends on what we are analyzing …

Year	1990	1995	2000	2005	2010	2015
Forest area in Colombia (km^2)	644 170	631 077	617 980	602 010	586 350	585 020

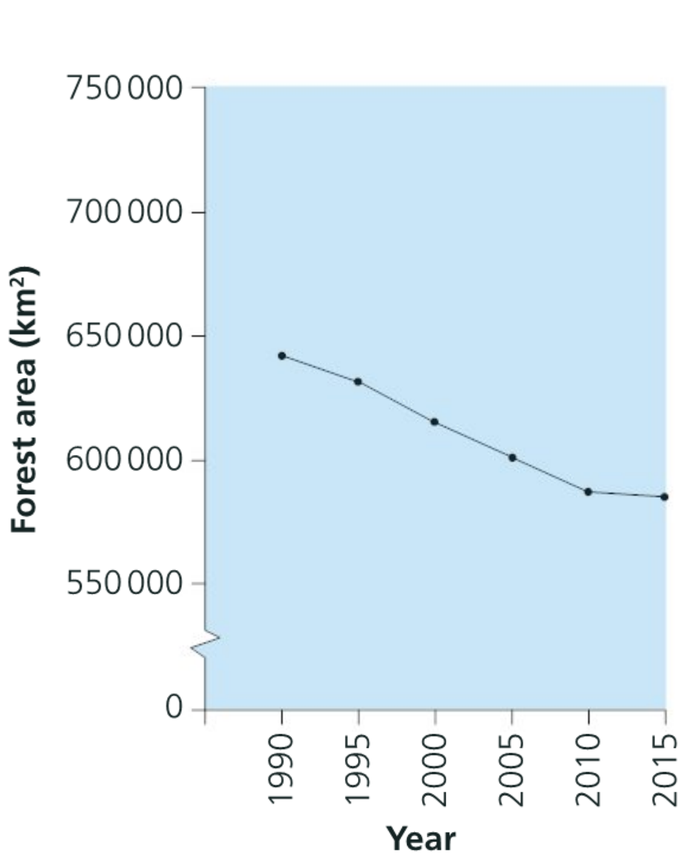
Source: World Development Indicators, the World Bank.

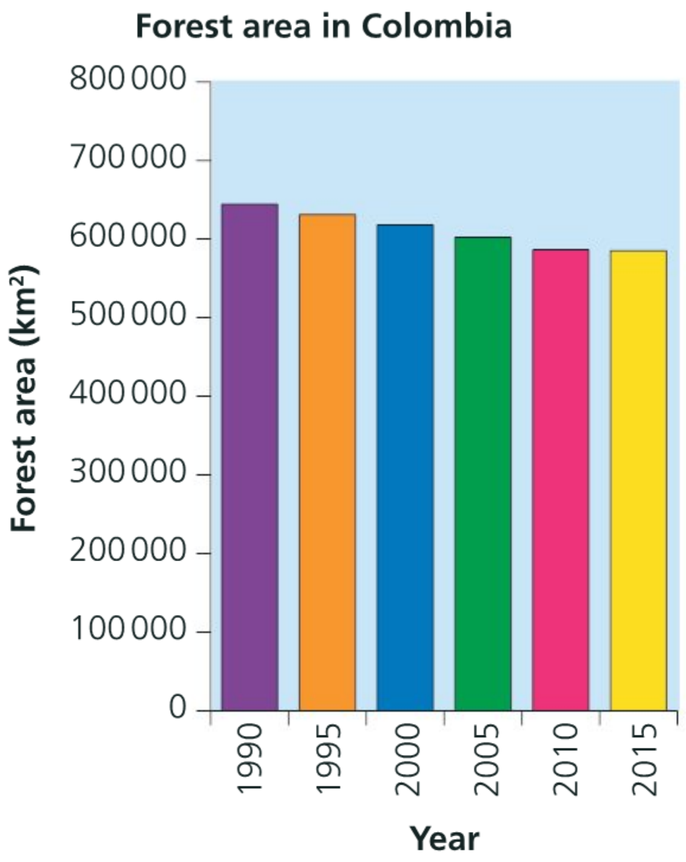
ACTIVITY: Representing quantitative data

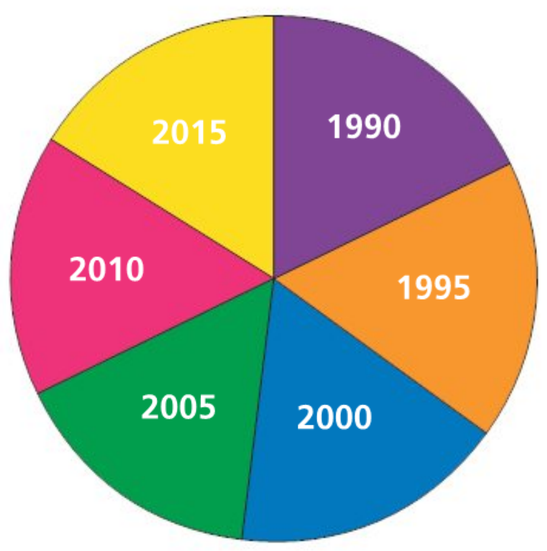
ATL

- Communication skills: Give meaningful feedback; Use appropriate forms of writing for different purposes and audiences; Make inferences and draw conclusions; Organize and depict information logically

Look at the diagram showing three different **representations** (numbers turned into pictures) of the same data in a line graph, bar graph/histogram and pie chart. Which one is useful in determining **trends** (patterns over time)? Why? Can you forecast the next few data points based on the trends you notice, and can you draw any conclusions?







Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding and Criterion C: Communicating.

Number of trash bags	Frequency (How many people had this number of trash bags?)
0	1
1	5
2	6
3	
4	
5	

As you may have noticed from the activity on page 61, it is important to choose the graphical representation that best suits your data's needs. For quantitative data, here are some questions to ask yourself:

- Do we want to compare numbers among, say, different populations? Or study ranges of numbers?
- Are we concerned with finding trends over time?
- Do we want to simplify very large numbers to make a comparison?
- Are we looking for a relationship between two variables?

Use a **histogram**.

Use a line graph.

Use a pictogram.

How about a scatter plot? (For more on scatter plots, see page 64.)

HISTOGRAMS

An urban street was surveyed to determine its level of garbage output in a week. Trash collectors were asked to note how many bags of trash each house left out on the most recent collection day. Here are the raw data.

3, 2, 2, 3, 1, 2, 1, 1, 3, 1, 2, 1, 2, 0, 2, 3, 3, 4, 4, 6

THINK-PAIR-SHARE

- **Why were the trash collectors asked to collect the data as opposed to surveying the residents?**
- **Who might be interested in this data? Why?**
- **Why would a histogram be an appropriate graph to represent this data?**

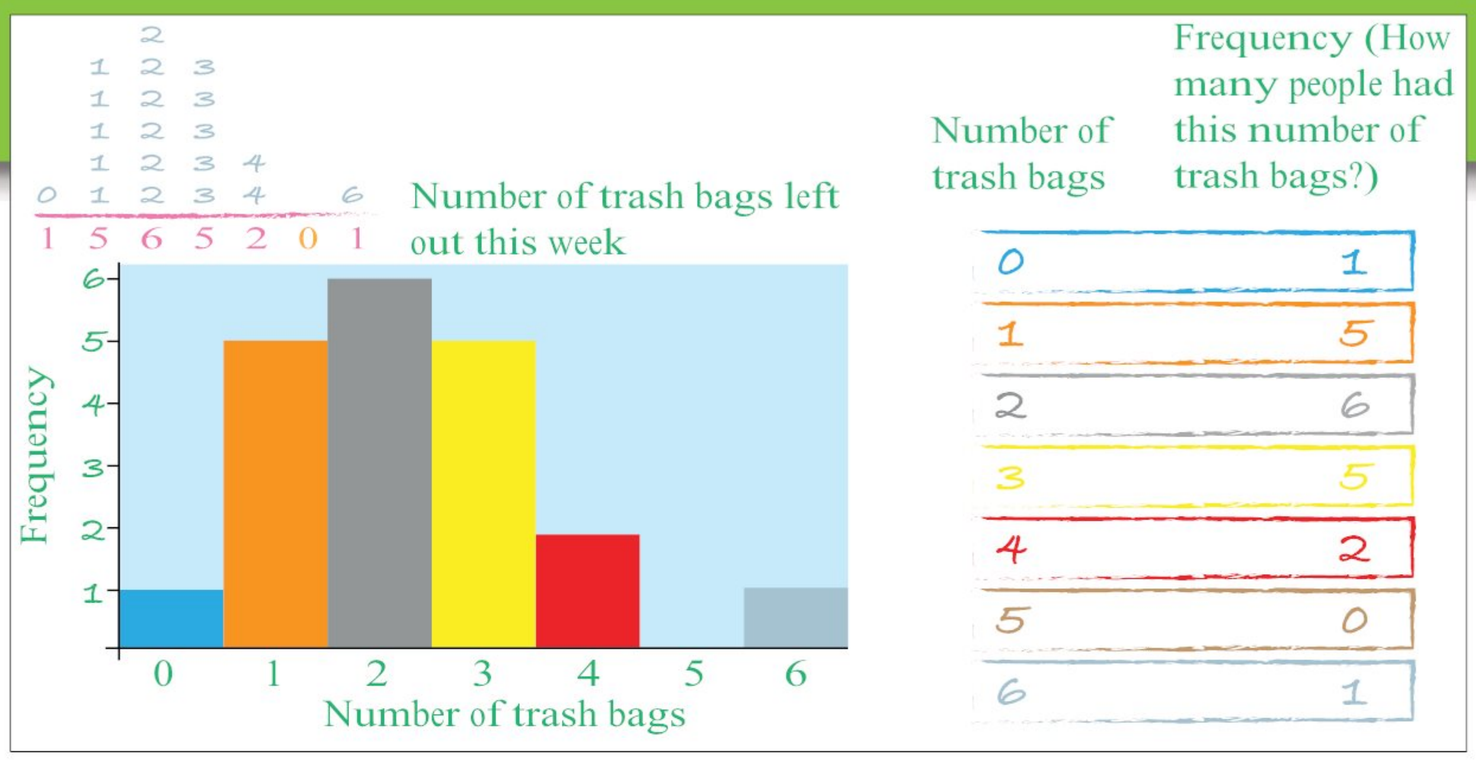
The data are valuable, but in their raw state they appear **messy**! Copy and complete the table to organize the data.

- How many houses (data points) are there in total?
- What do your frequencies add up to?
- How can you ensure you have accounted for each household in your table?

Drawing histograms

Now we are ready to draw! Look at this guide on how to draw a histogram.

- First we set up our axes.
- What is the variable we are interested in? This goes on the x-axis.
- What are the highest and lowest numbers the variable takes on in our data? Do you have enough space to count by ones? This will help you determine your scale.
- In a histogram, the frequency is always listed on the -axis so that the height of the bars gives the popularity of that number (in this case, the number of garbage bags per household).
- Now pick a descriptive title for your graph. Good communication is the key to an effective graphical representation. Keep it simple, but ensure you are clearly communicating your variable and where your sample came from.
- Something interesting a histogram can show us that a bar graph can't is the **spread of data**. Is the data fairly consistent and hovering around the same few numbers, or is it spread across a wide range of numbers? The width of the graph shows us this instantly.



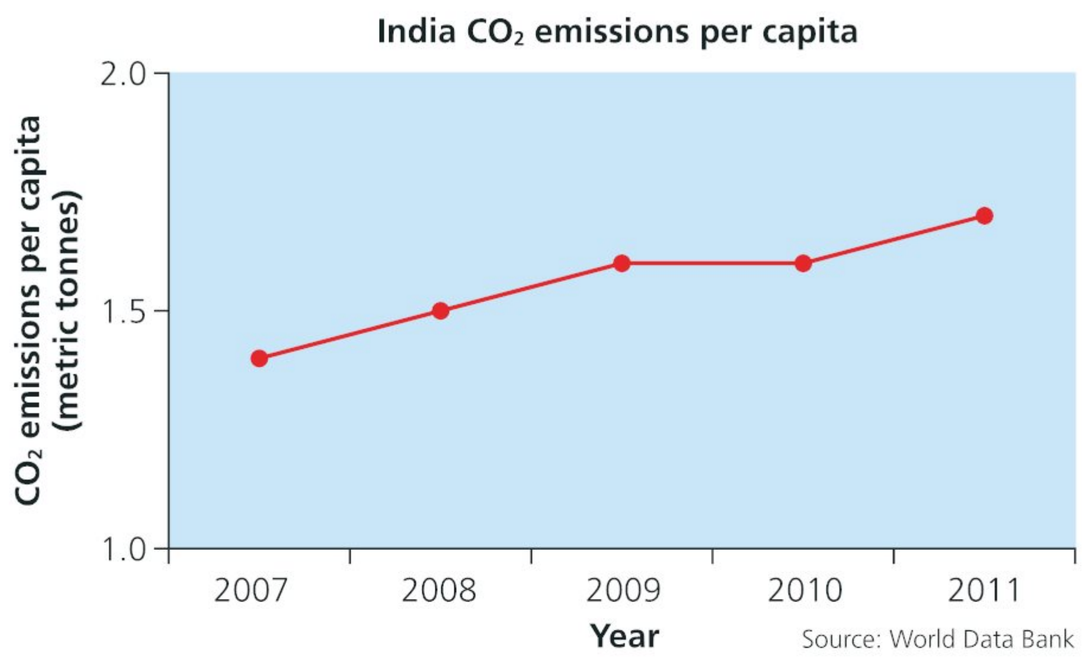
Hint

Label, label, label! Whenever you are drawing a graph, ensure you have labelled:

- *x*-axis title
- *x*-axis scale
- *y*-axis title
- *y*-axis scale
- graph title
- any colour key with a legend as necessary.

LINE GRAPHS

The graph shows the carbon dioxide emissions released per capita in India over the past 15 years. We'd like to determine the overall trend (have emissions increased or decreased?), as well as identify any sudden changes (dips or spikes) that may have occurred in that time. A line graph is an excellent way to display and identify trends over time.



Why 'per capita'?

Imagine you are working as part of a team of 5 people to collect donations of non-perishable (lasting a very long time) foods for a local food bank. As an incentive, the project leader offers a pizza lunch to the team which collects the most donations by the end of the week.

Your friend Louis' team of 27 students wins the pizza lunch. They collected 54 cans and your team collected 50. Does this seem fair to you? Why or why not? What would be another way to determine the winners of the pizza lunch? Would your new approach be more or less fair?

Per capita literally means 'per head'; as every person has a head, it translates to 'per person'. In this example, your team collected an average of $\frac{50}{5} = 10$ cans per person, whereas Louis' team collected $\frac{54}{27} = 2$ cans per person.

Because population can significantly affect results, data are often translated into a 'per capita' number. A heavily populated country will naturally produce more emissions than a less populated one, so a reasonable way to compare two such countries is to calculate the emissions per capita – how much gas is being emitted into the atmosphere **per person**. This will give us a better picture of how well a country is managing its carbon waste.

ACTIVITY: An end to extreme poverty?

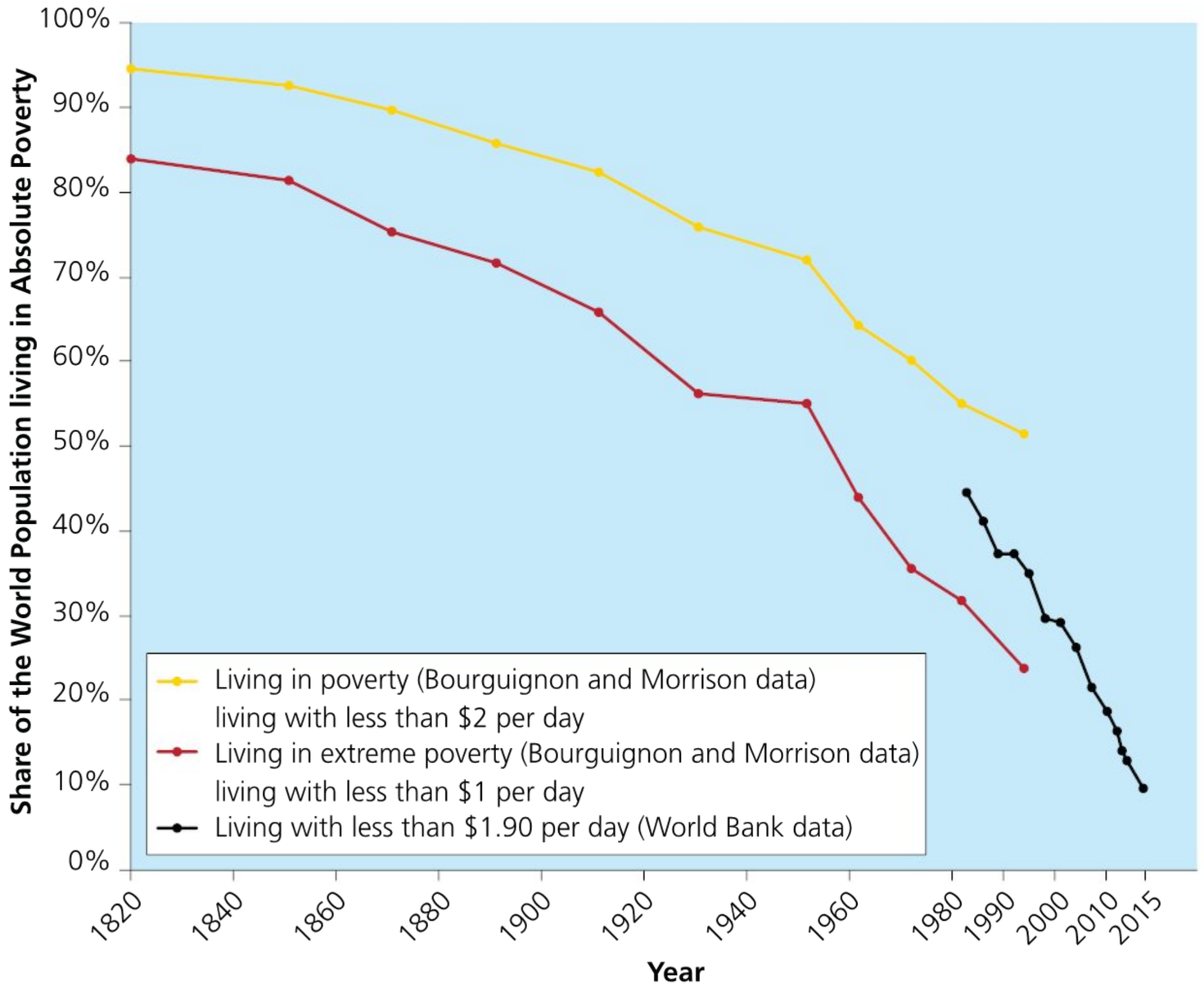
In November 2014, the UN General Assembly posted the first target of its 17 Sustainable Development Goals:

- **In 2030, to eradicate extreme poverty for all people everywhere, currently measured as people living on less than $1.25 a day.**

Can this be possible? How can this graph **justify** such a target?

Share of the World Population living in Absolute Poverty, 1820–2015

All data are adjusted for inflation over time and for price differences between countries



Data sources: 1820–1992 Bourguignon and Morrison (2002) Inequality among World Citizens, In the *American Economic Review*; 1981–2015 World Bank (ProvcalNet)

SCATTER PLOTS

Bar graphs and pie charts are terrific ways to present uni-variate data visually, but if we are trying to compare **two** variables at the same time, or to see if and how they might affect one another, we turn to scatter plots.

In a scatter plot, every element of the data is represented by a point: for instance, every person you surveyed, every animal you observed, every country you collected secondary data about, every crime committed and recorded, and so on. The placement of a data point on the graph works in the same way as the placement of a point on a map.

Similarly, in a scatter plot, we place a data point where the quantities of two variables meet.

In the scatter plot on the next page, secondary data was gathered about each country and each country is represented by a point. Laos was found to have a Gross Domestic Product per capita (GDPPC) of $5076.22 (international dollars) with 29.95% of its population in extreme poverty. We find the $5000 mark on the x (horizontal) axis, and the 30% mark on the (vertical) axis, and draw our point where they meet, or converge.

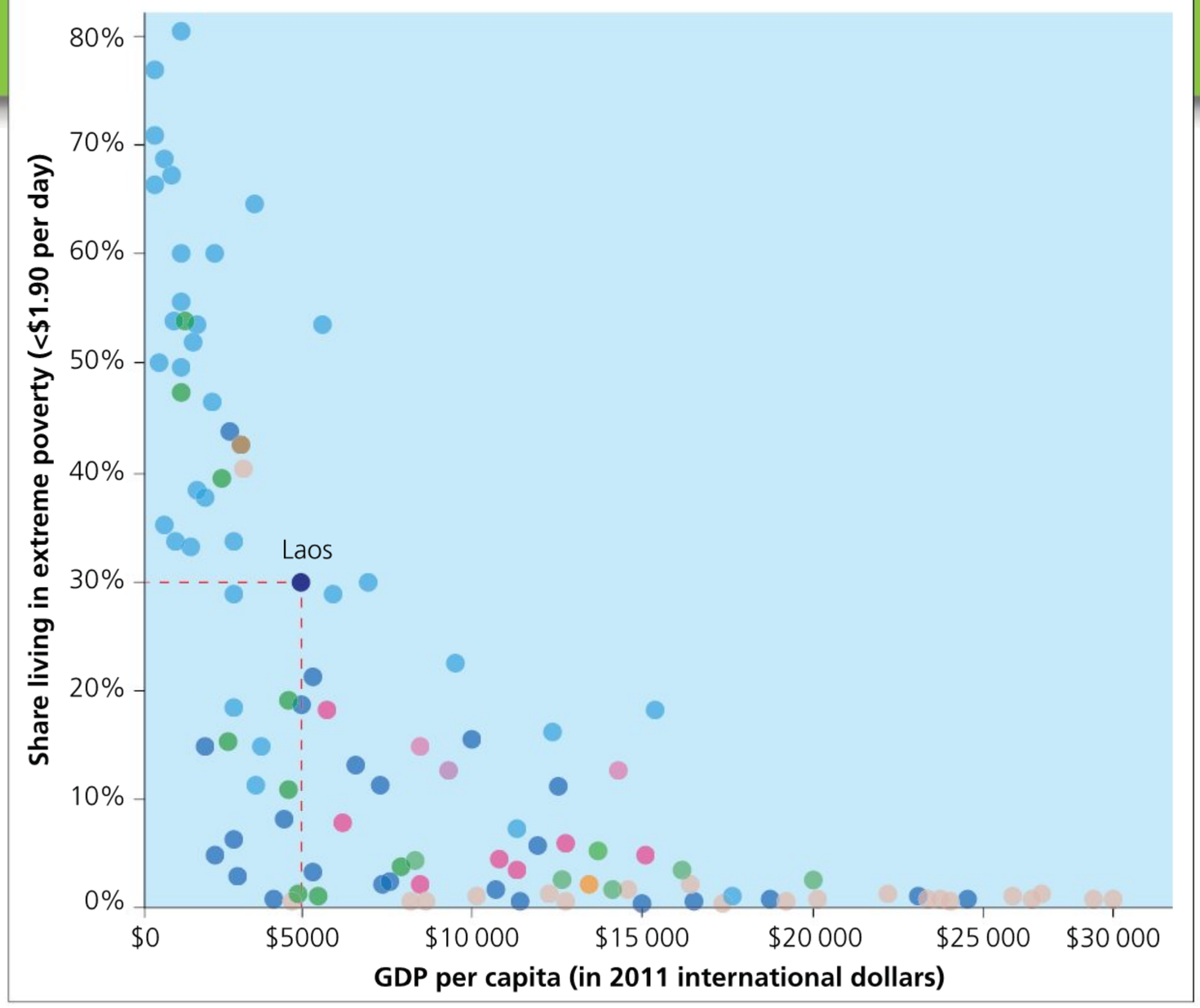
How might you determine the total number of countries that were included in this graph?

Looking more generally at the graph, you may notice many points at the top left, and many at the bottom right. What might this imply about the relationship between our two variables? When one of them is high, the other is low and vice versa. This means that a country with a low Gross Domestic Product per capita (GDPPC) has high poverty rates, and a country with high GDPPC has low poverty rates. If we wanted to draw a conclusion, we could say 'the richer the country, the more it takes care of its people'.

What does the graph tell you about how the world might end extreme poverty? How do you think we might go about doing that?

The share of people living in extreme poverty vs GDP per capita, 2008 to 2014

Both measures are adjusted for inflation over time and for price differences between countries and are expressed in 'international dollars'. Extreme poverty is defined as living with less than $1.90 per day.



Links to: Individuals and societies

When orienting on a map, we determine two variables – longitude and latitude. We locate each dimension separately on the map, and run one finger along each until those two particular lines meet.

ACTIVITY: Coincidence? I think not! (but I could be wrong)

ATL

- Information literacy skills: Finding, interpreting, judging and creating information

Sabermetrics is a statistical method for analyzing data in baseball performance. It is used to make play decisions and to decide which players to hire. Its pioneer, Bill James, used sabermetrics to help the Oakland A's sustain incredible success on a limited budget. This was so impressive, a book and later a movie 'Moneyball' chronicled it. These methods for appraising players soon spread into other sports. Many football fans were shocked in 2016 when Leicester City became Premier League champions, and interestingly they had one of the most advanced use of analytics in the league.

What if you worked for Leicester City and wanted to come up with a new variable that affects a forward's performance? (A forward's primary function is essentially to put the ball in the back of the net – to score the goals.) To keep things simple, let's use 'number of goals' as the variable that establishes how well a forward performs. Now your job – to come up with a second variable that might affect his number of goals. For instance, your variable could be 'number of red cards' (penalties). Does the number of a forward's red cards decrease his number of goals, due to less play time on the field, or does it increase them due to his aggressive play?

Decide on the variable you wish to **explore**. Use this search term: **Premier League players**. Then find 20 forwards in the league and record their number of goals, and the number that corresponds to your chosen variable. Create a scatter plot, such that each point represents one of your 20 players. Is there a correlation? **Justify** your response by referring to your graph. If yes, why do you think this correlation exists?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

BAR GRAPHS

ACTIVITY: Sticky problem

ATL

- Communication skills: Give meaningful feedback; Use appropriate forms of writing for different purposes and audiences; Read critically and for comprehension; Make inferences and draw conclusions; Use and interpret a range of discipline-specific terms and symbols; Understand and use mathematical notation; Organize and depict information logically; Structure information in summaries, essays and reports

The bar graph on the next page compares the 10 most devastating oil spills in history. As in every bar graph, the **nominal** (qualitative) data – the names (of places, colours or people, for example) – are on the x-axis and the measurement is along the -axis.

We are going to rewind the process of creating a bar graph by filling in a copy of the table based on the data you see in the graph.

Name of oil spill	Amount of oil spilled (rounded to the nearest 1 million kg)

If you were trying to convince a world leader not to set up an oil rig in your hometown and needed impactful evidence to back up your arguments, would you use the table or the graph? Why?

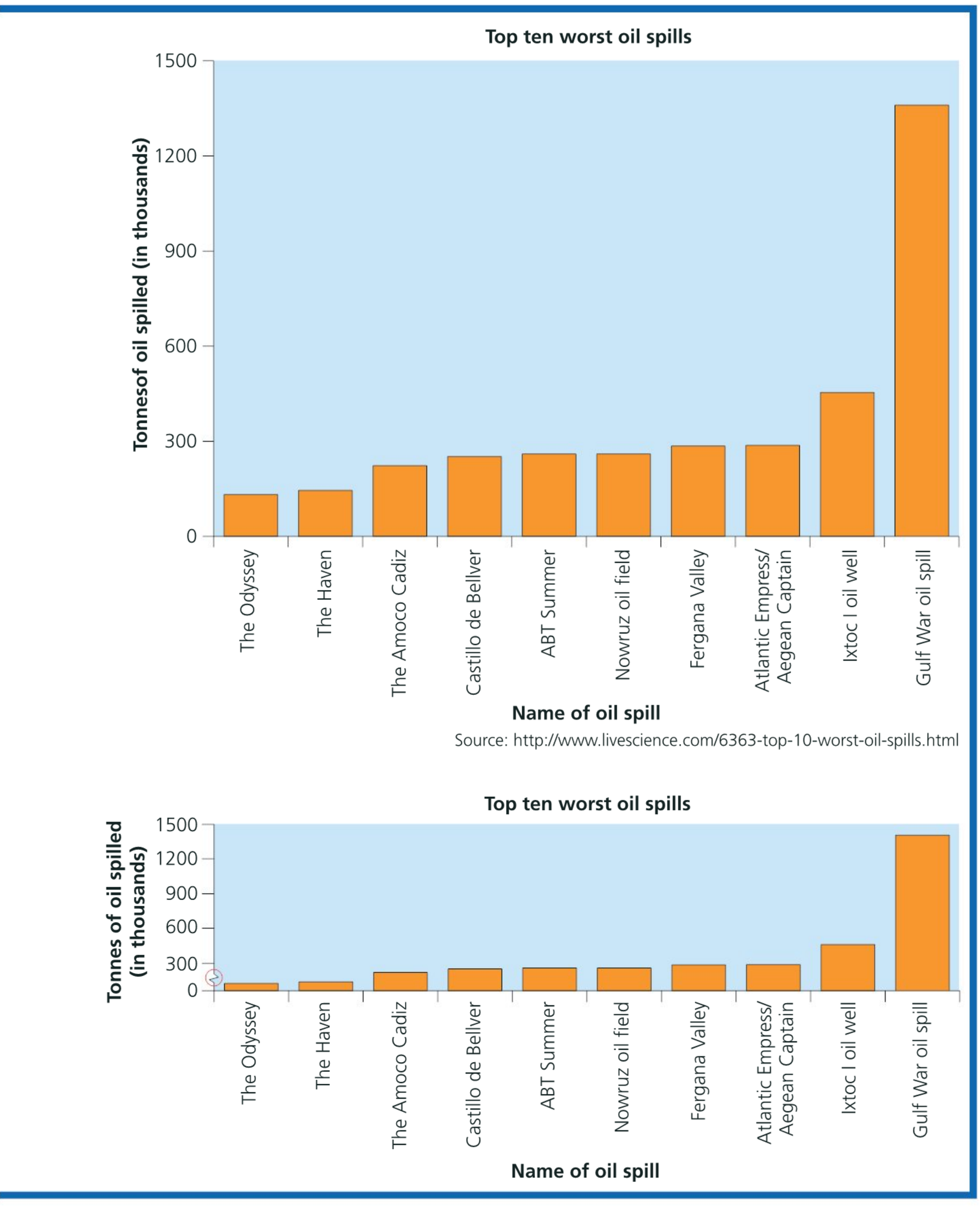
The lower diagram opposite shows the same graph with a little cheat tactic: when the smallest measurement in your data is a large number (in this case, 120 million kg), it can be tedious and take up a lot of space to begin your scale at zero. You have the option of using a 'break' in your graph (circled) – this tells your reader you have started at zero, then skipped up to a particular number and resumed from there. Breaks can only be applied directly above zero.

In the case of wanting to convince an audience of the gravity of oil spills, which bar graph would you rather show and why?

Why is it crucial that your scale counts by the same number interval throughout, and that it is divided equally along the axis line?

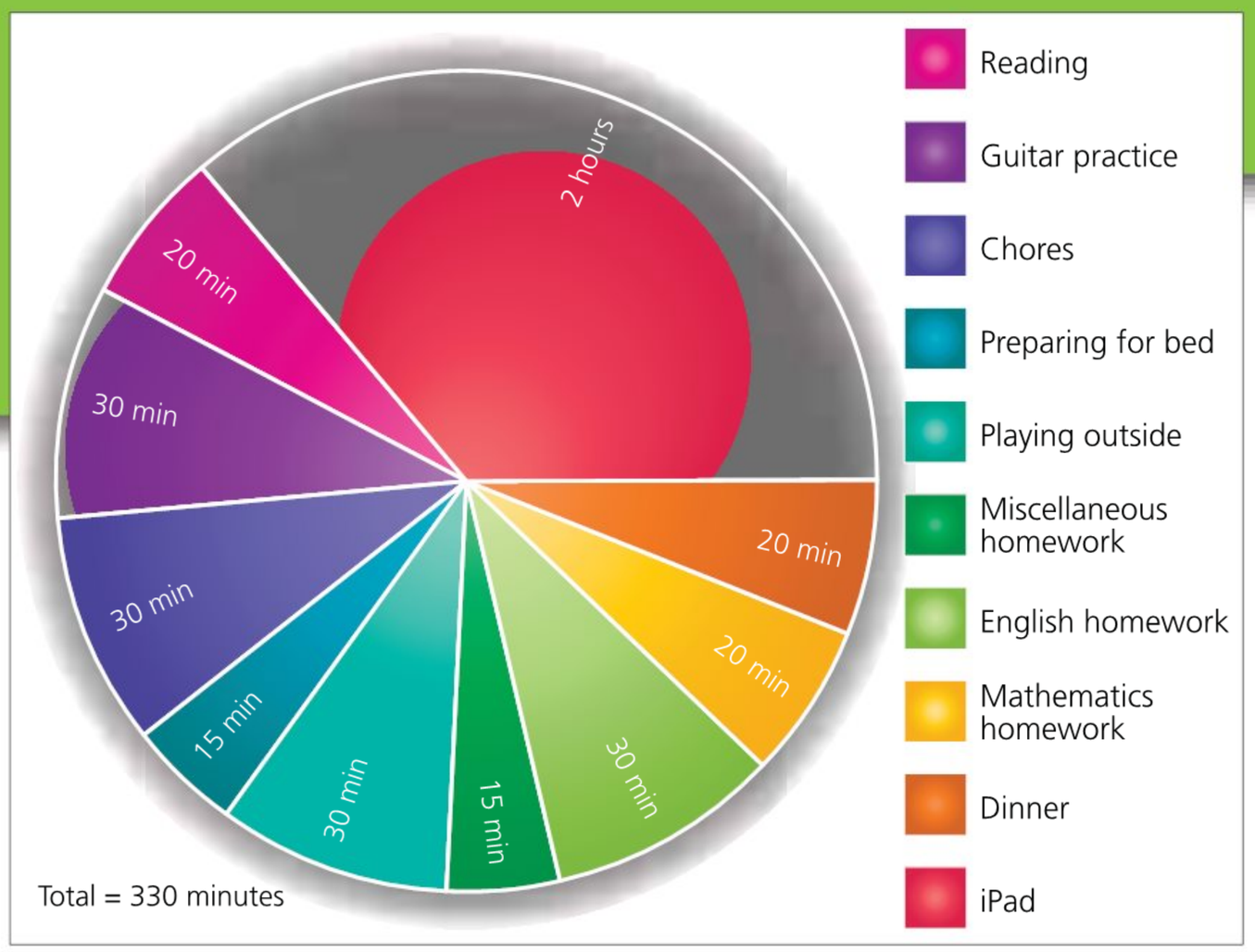
Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.



EXTENSION

How might you represent negative figures in a bar graph? In what situation might you need to do this? Give two examples.



PIE CHARTS

Shamar spends 2 hours of his 5.5 hours of free time after school on his iPad. It may not seem like a lot of time – one show and a few social media updates – but when he sees this represented visually, he reflects seriously on his priorities.

What percentage of his free time does Shamar spend on his iPad? What percentage of his free time does he spend reading?

Shamar spends 2 out of 5.5 hours on his iPad.

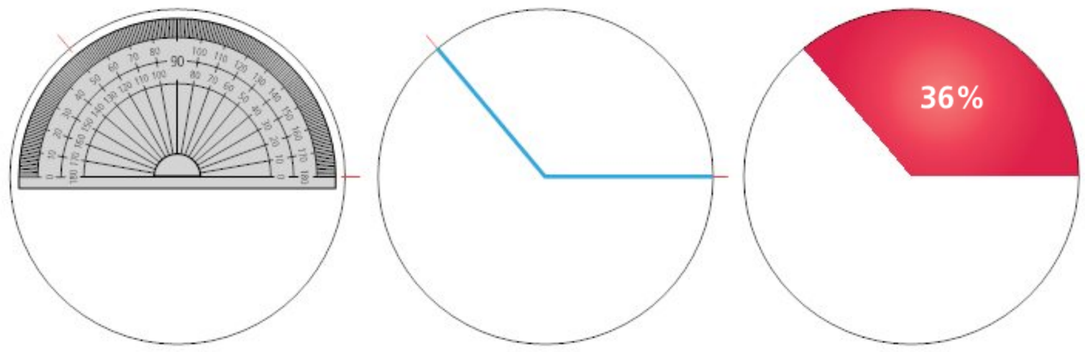
$2 \div 5.5 = 36\%$

How do we shade in 36% of a circle?

A circle has 360 degrees. To shade in 36% of it, we need to calculate 36% of 360, as shown on the right.

To display this information in a pie chart, we draw a radius anywhere in the circle, find 130 degrees from that line, and shade in that area.

I prefer decimals:	I prefer fractions:
Change 36% to a decimal: 0.36	Change 36% to a fraction: $\frac{36}{100}$
$0.36 \times 360 = 129.6$ (approximately 130°)	$\frac{36}{100} \times 360 = \frac{12\,960}{100}$ or $\frac{1296}{10}$ (approximately 130°)



ACTIVITY: Your free time in a pie chart

ATL

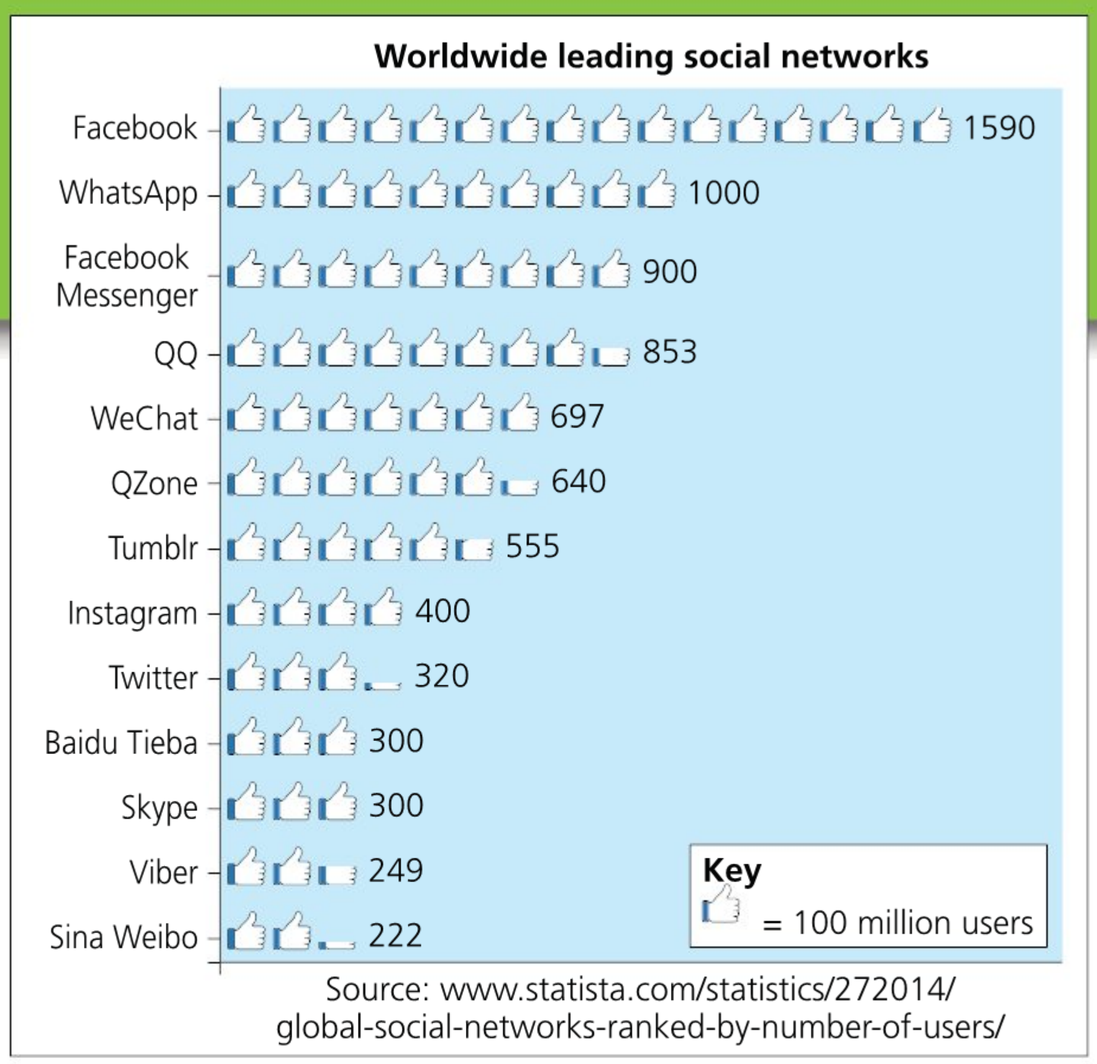
- Reflection skills: Keep a journal to record reflections; Consider personal learning strategies
- Information literacy skills: Collect, record and verify data; Process data and report results

Now it's your turn! This evening, keep a log of everything you do after school and how long you spend doing it. (Be honest!)

Start recording when you get home and stop when you finally get to bed. You might use a stopwatch function on a phone and have each activity recorded as a new 'lap'! You could even use a real stopwatch for the same function.

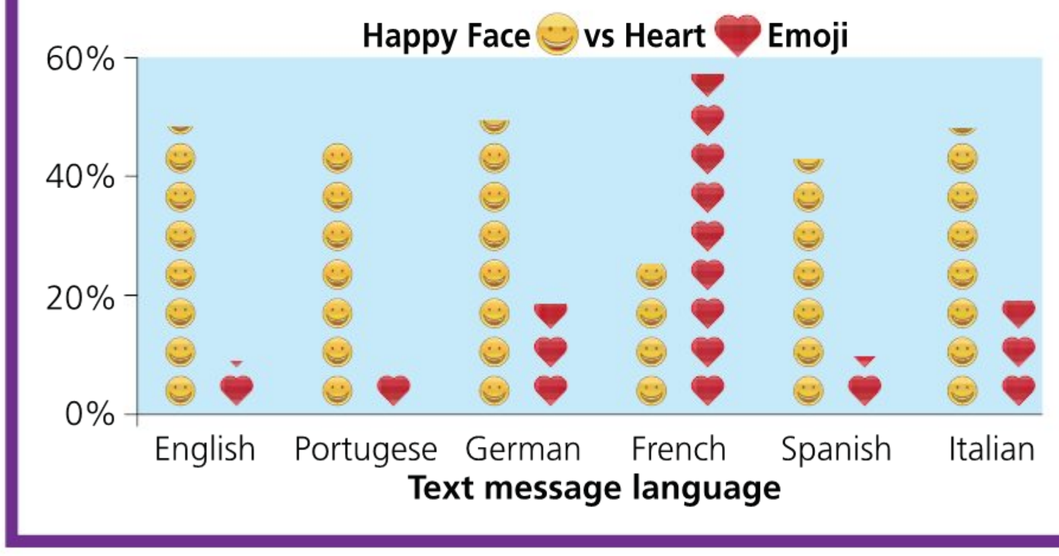
Then **draw** a pie chart to visualize your free time.

Copy and complete the table opposite to help you organize your information.



EXTENSION

The pictographic below shows the frequency of the two most frequently used emojis in texts of various languages. It is missing a crucial piece of communication – yes, that's right, the key! Can you determine what percentage each individual emoji represents? Was this representation well chosen? **Justify** your response.



PICTOGRAPHS

If your data have a large difference between the highest and lowest values and a bar graph is too cumbersome, you may consider a pictograph. You may also consider this presentation if you want a fun display.

For example, let's take a look at which social media forums are used most worldwide.

1. First, **select** a suitable social media icon.
2. Next, decide on a number that the icon will represent.
3. Finally, **draw** your axes.

In our example, we chose the 'like' icon from facebook – it will represent the number of people who like each social media app.

Our highest number is 1590 million and our lowest is 320 million, with all but one number under 1000 million. If we count by millions we won't have enough space and if we count by 500 millions, the majority of our data will be too close together to notice any differences. So, let's try using each icon to show 100 million.

Decide on a scale (something a little bigger than the number your icons represent) and remember to label!

Activity	Time spent	Fraction/percentage of evening	Degrees in a circle

Reflect on your use of time

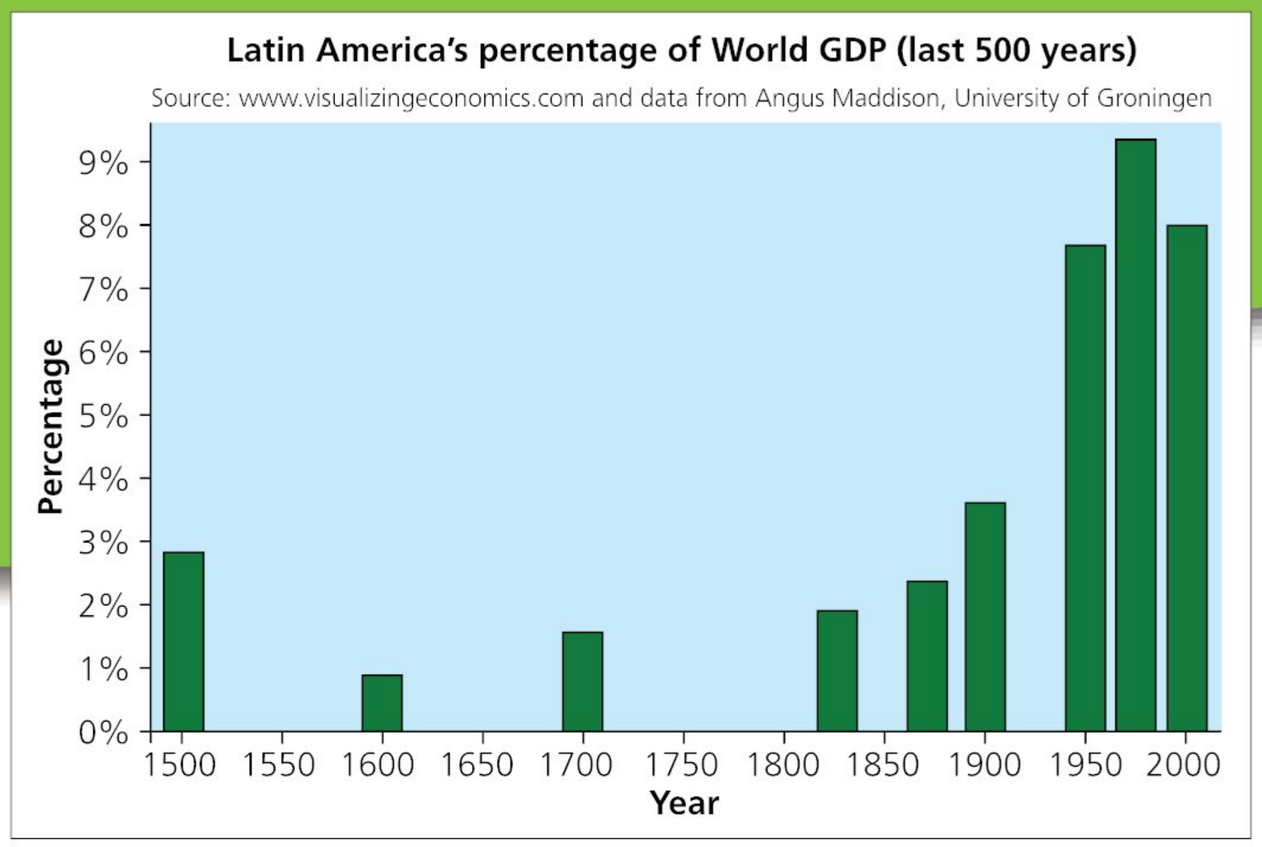
Seeing this visual representation of your evening, are you surprised by anything? Proud? Disappointed? Anything you would like to change?

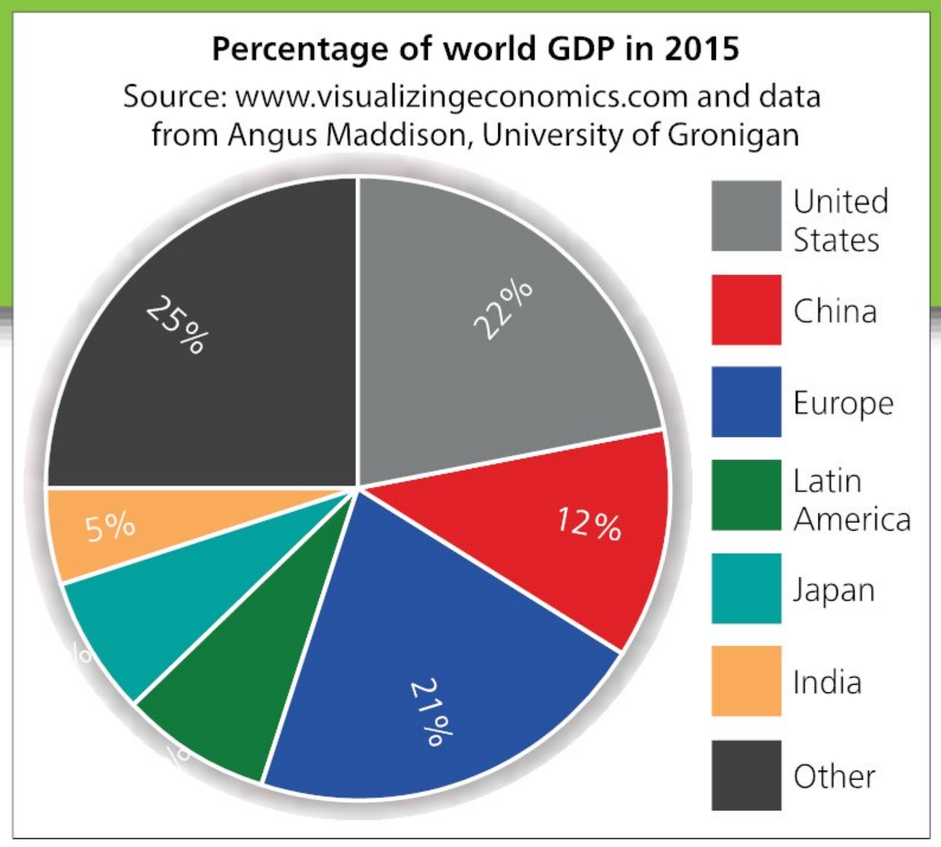
Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

EXTENSION

A circle is the norm for pie charts. Any other shape would not be a 'pie', but other shapes can work the same way. Recreate your pie chart using a different shape. This could be anything from a regular polygon to a silhouette of an animated character or logo.



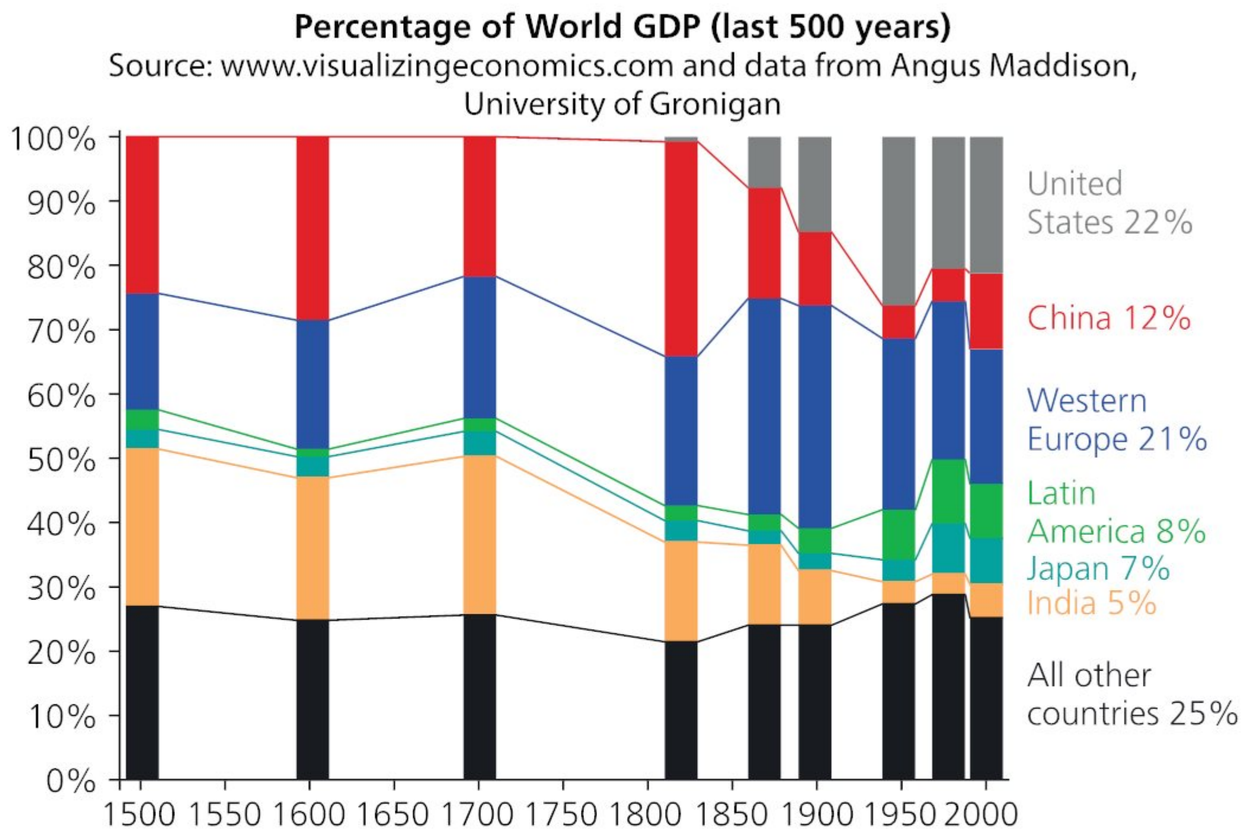


HYBRID GRAPHS AND CHARTS

How do we distinguish a rich country from a poor one? Many people look at the country's Gross Domestic Product per capita – its GDPPC. If we were to take all the money every citizen earned from sales of goods and services, subtract the country's expenditures, then share what's left equally among everyone – this amount is the GDPPC.

Above are two graphs – a bar graph of Latin America's percentage of the world's GDP over 500 years, and a pie chart of the world's GDPPC in 2015 split by continent.

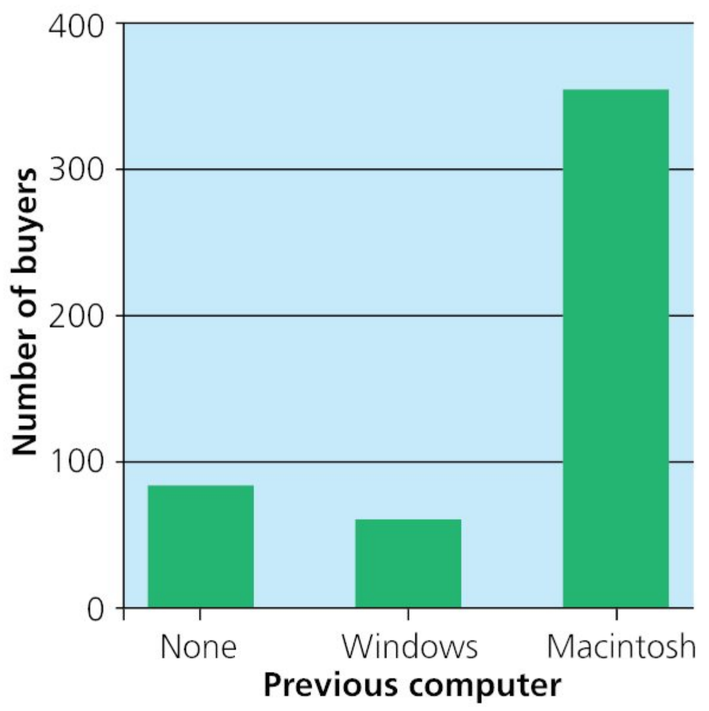
If we wanted to show this data simultaneously (assuming we had corresponding bar graphs for other regions and similar pie charts for different years), we could use a split bar graph as shown below:

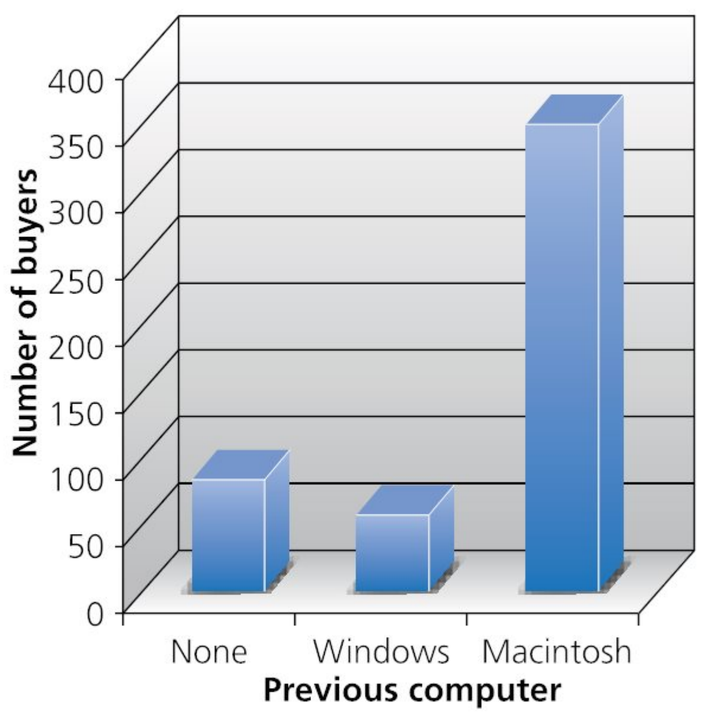


Sometimes we like to over-decorate our graphs to make them more appealing, but always remember what you are trying to achieve with your graph – you want an accurate representation of your data. Take a look at the two graphs on the right. Which do you think gives the more accurate presentation of the data?

Hint

The KISS principle is to **keep it short and simple** or **keep it simple and straightforward**.





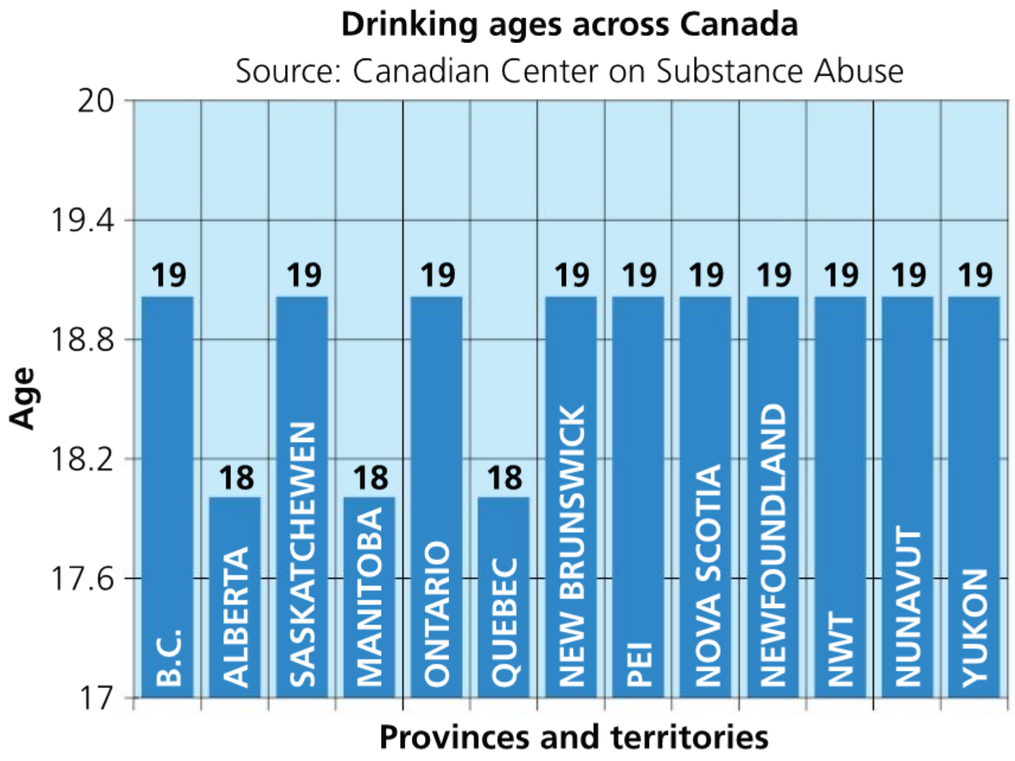
Hint

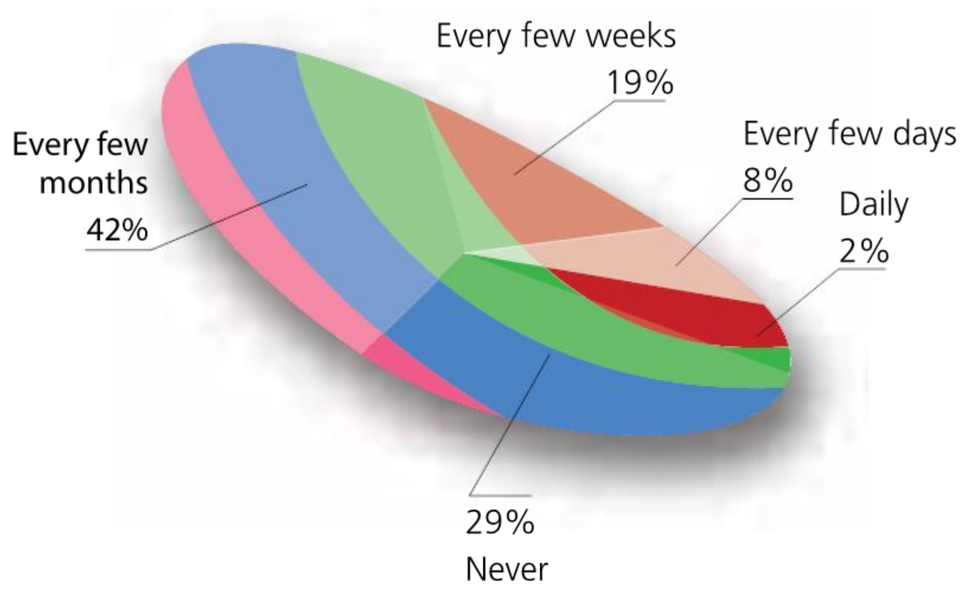
What is misleading about a split bar or 3D bar graph?

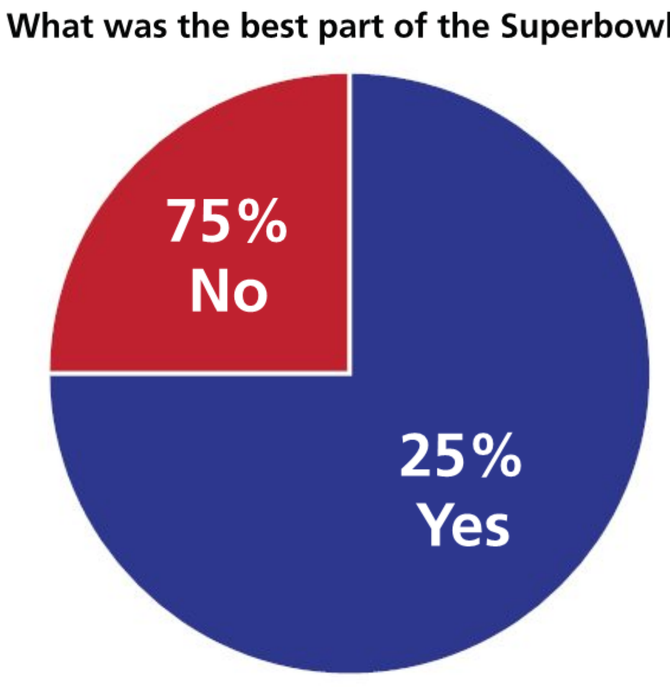
Can graphs lie?

ACTIVITY: Graphs gone wrong!

Each of these graphs has been adapted from real media. They really were included in marketing brochures, newspapers and on TV! **Identify** what is wrong with each.







ACTIVITY: Home and away

ATL

- Information literacy skills: Collect, record, and verify data; Access information to be informed and inform others; Present information in a variety of formats and platforms; Process data and report results; Create references and citations, use footnotes/endnotes and construct a bibliography according to recognized conventions; Identify secondary sources

Your task is to compare interesting data from your country with those from another country of your choice. Remember, we are focusing on 'Globalization and sustainability', so try and think of some variables that link to this topic. (Examples could include clean water, recycling rates, proportion of farmland, social innovation, air quality and garbage output.)

You will need to choose different data sets to show you know how to **apply** the following presentations:

- **Line graph**
- **Pie chart**
- **Bar graph (bonus for split bar graph)**
- **Pictogram**
- **Scatter plot**

Include a short description for each graph, including why you chose that style to present your data. In your description, say:

- **if your data are qualitative or quantitative**
- **if your data are primary or secondary**
- **the aim of your graph**
- **any patterns or trends you noticed**
- **your source**
- **any other findings you feel are interesting.**

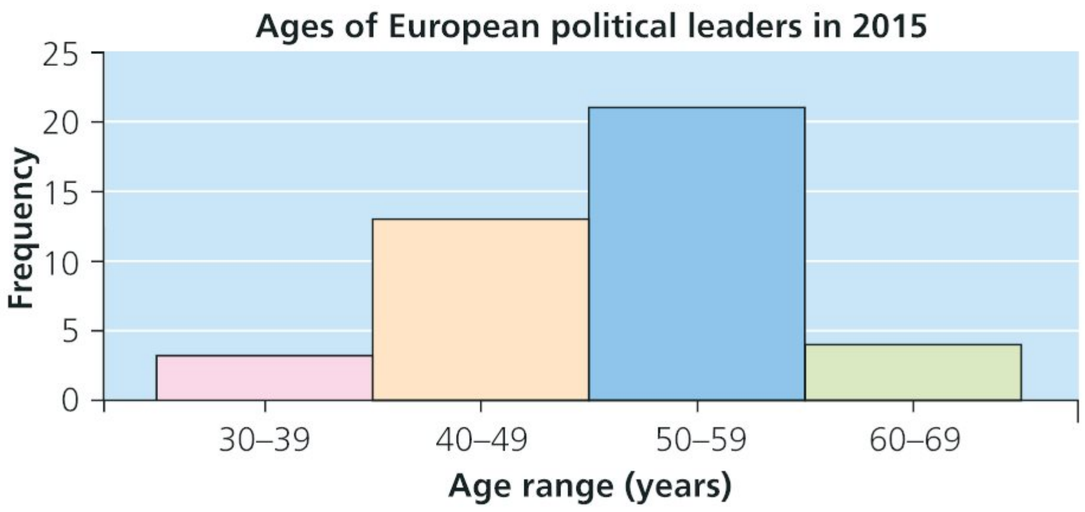
Remember to include graph labels and titles, plus a key if necessary.

Assessment opportunities

- This activity can be assessed using Criterion C: Communicating.

What kind of difference can one person make?

MEASURES OF CENTRAL TENDENCY



The histogram shows the age ranges of the various European political leaders. Can we determine the **three measures of central tendency** from this graph?

The **mode** is the most recurring number. In other words, it is the number that came up most frequently in the data – this translates to the tallest bar in the histogram. Ages 50–59 (the blue bar) came up 21 times. This is the most frequently occurring number, and therefore our mode.

The **median** is the middle number in the data range. Looking at the histogram, to find the middle number, we first add the frequencies for the bars together to determine the total number of countries from which the data has been collected: 2 + 13 + 21 + 4 = 40

Mean, median, mode (MMM)

Recall that, when we generalize large amounts of data into a single number, we are usually referring to either the **mean**, the **median** or the **mode**. Together, these are referred to as the **measures of central tendency** – meaning 'how they tend to the centre', or group around a central value.

If there are 40 data points, then the median will be the twentieth data point.

Age range	Frequency
30–39	2
40–49	13
50–59	21
60–69	4

These are the first two data points.

Rows 1 and 2 make up the first (2 + 13) data points; we are looking for the twentieth data point.

The sixteenth to thirty-sixth data points lie here, which means the twentieth data point belongs to this interval.

Source: http://chartsbin.com/view/38561

Since the twentieth data point lies in the 50–59 age range, this age range is our **median** (mid-point).

To find the **mean**, we need to calculate the average age. Typically, the mean is the sum of all the data points (in this case, the ages) divided by the sum of data points (here, the number of political leaders being studied). Referring to the frequency table above, it is impossible to list the leaders' exact ages as we only know the age **range** of each person sampled. Instead, we will use the **median** of each age range as our best estimate.

Mid-point of age range	Frequency
35	2
45	13
55	21
65	4

Which allows us to calculate the mean:

$$\frac{35 + 35 + 45 + 45 + 45 + 45 + 45 + 45 + 45 + 45 + 45 + 45 + 45 + 45 + 45 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 65 + 65 + 65 + 65}{40} = 52$$

THINK-PAIR-SHARE

Give three examples of real-life instances in which the mean is the best measure of central tendency to use, then repeat for median and mode.

What sorts of stories can data tell us?

ANALYZING DATA AND DRAWING CONCLUSIONS

Data tell us things beyond averages and correlations. Among these things is the range or **spread**.

How consistently do the data we found occur? Are they very random? In which case, our measure of central tendency isn't a very reliable one. Or are they extremely consistent, so we may count on our findings occurring again and again?

DISCUSS

Why can't we discuss the spread of data when looking at a bar graph?

RANGE

One of the quickest ways to determine whether or not your data are consistent is to calculate their **range** – the difference between the highest value, the **maximum** and the lowest value, the **minimum**.

range = max – min

This is like finding a distance. 'How far do I have to travel to get from one end of my data to another?' The further you have to travel, the more spread out (and less consistent) your data are.

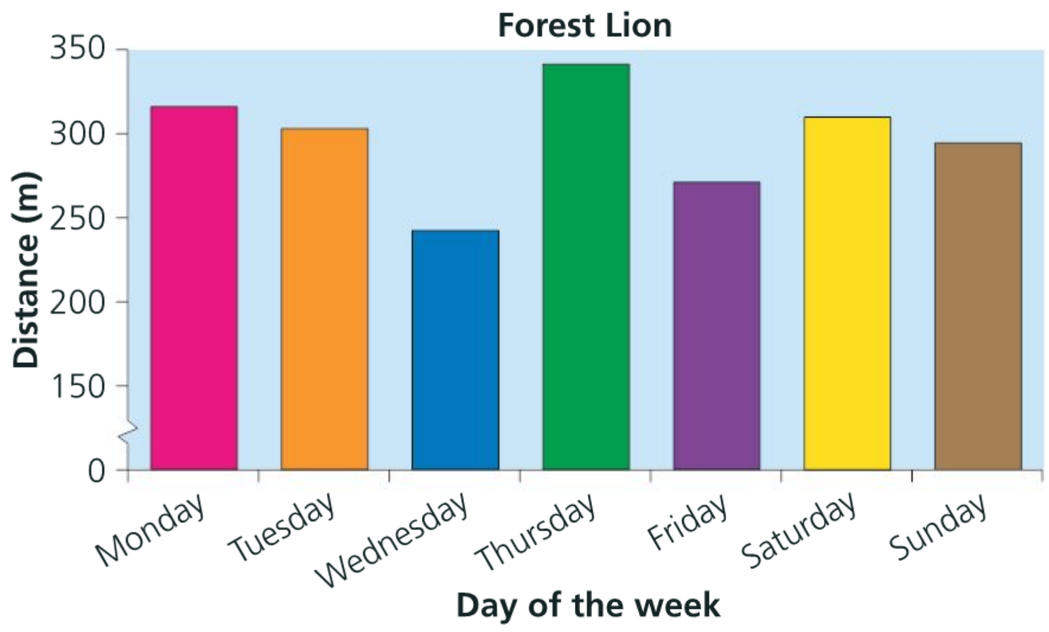
ACTIVITY: Who is the better driver?

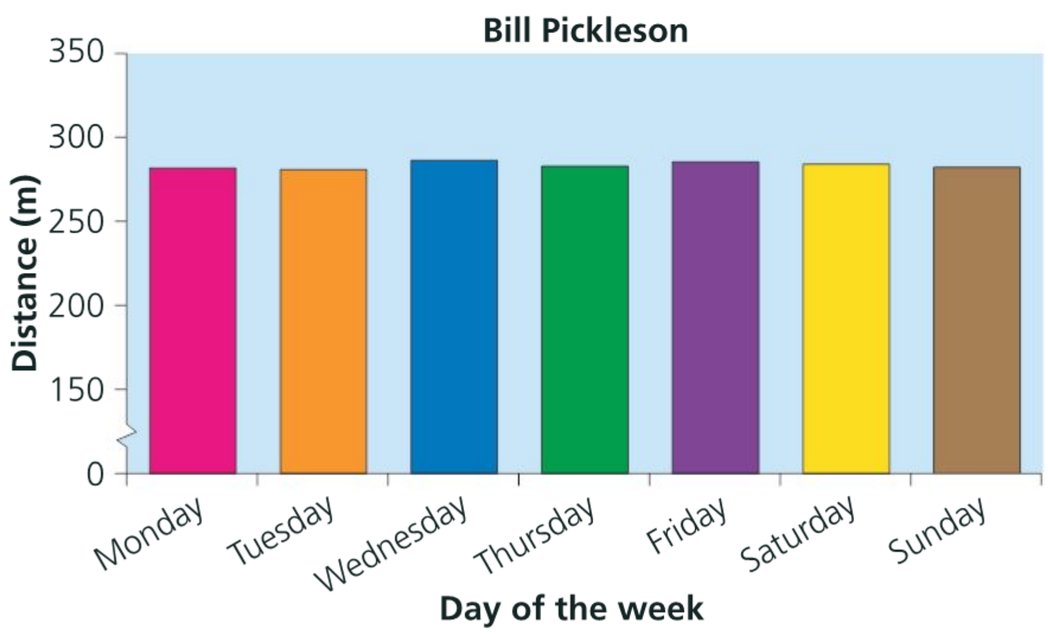
ATL

- Critical-thinking skills: Interpret data; draw reasonable conclusions and generalizations
- Transfer skills: Apply skills and knowledge in unfamiliar situations

The first shot at every hole in golf is the drive. Golfers go to driving ranges to practise this shot over and over. The bar graphs display the average drive of two different professional golfers every day for a week. Which golfer is the better driver?

1. **Calculate the weekly mean for each golfer.**
2. **Discuss the difference between the two graphs.**
3. **Who is the better driver?**





Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

In statistics, outliers are any data points that don't follow the general trend. People across many professions devote a lot of effort to analyzing outliers and what causes them.

'You know how you hear someone say of Bill Gates or some rock star or some other outlier—'they're really smart' or 'they're really ambitious'? Well, I know lots of people who are really smart and really ambitious, and they aren't worth 60 billion dollars. It struck me that our understanding of success was really crude—and there was an opportunity to dig down and come up with a better set of explanations.'

Author Malcolm Gladwell, on what inspired him to write his book, Outliers.

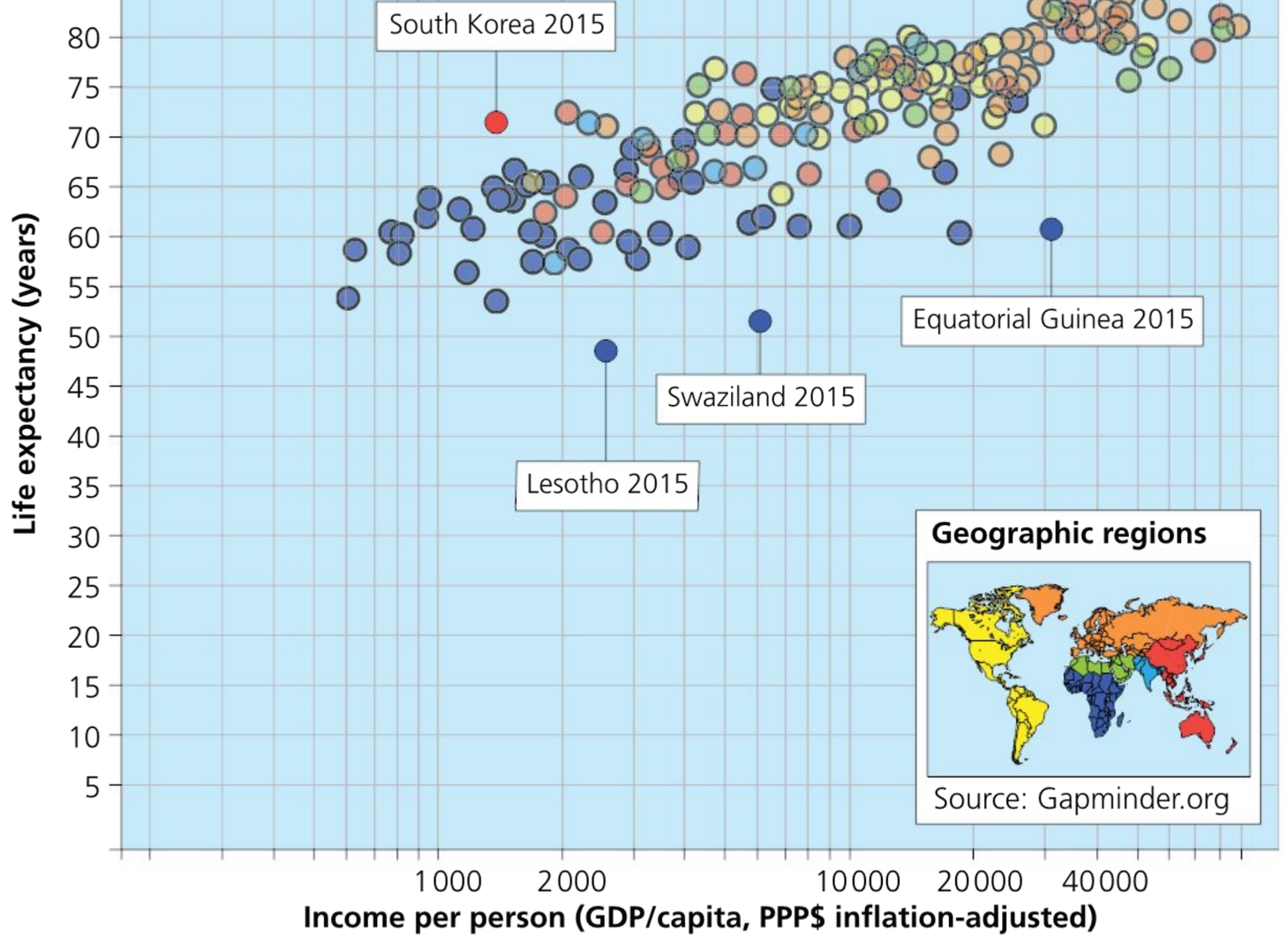
OUTLIERS

This scatter plot explores how a country's economy affects its health by examining the relationship between income per person and life expectancy. (PPP means purchasing power parity or putting currencies on an equal level, adjusting so that purchasing power is the same in all countries.) In general, the graph is trending upward, meaning that as wealth increases, average life expectancy rises as well. There are, however, a few outliers. Can you identify a few points that don't seem to be following the trend?

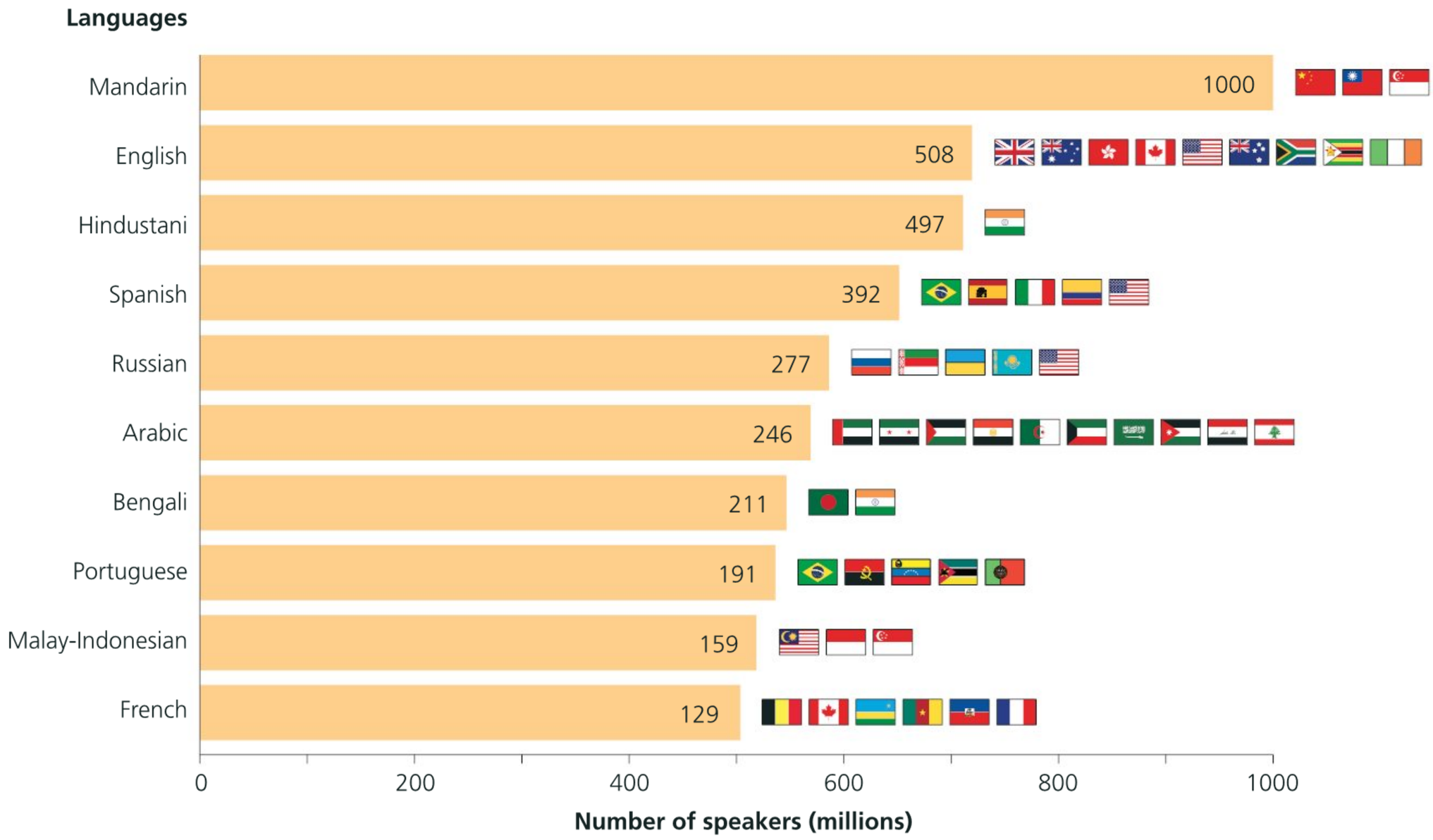
Looking at the key, you can see which continents the outliers come from. Can you think of a reason these countries might be off-track from the rest of the world?

Opposite, the countries that make up the major outliers are revealed. Research to see if you can explain why South Korea has an unusually high life expectancy, and why Lesotho, Swaziland and Equatorial Guinea seem to have poorer health despite their economic standing.

The global relationship between income per person and life expectancy

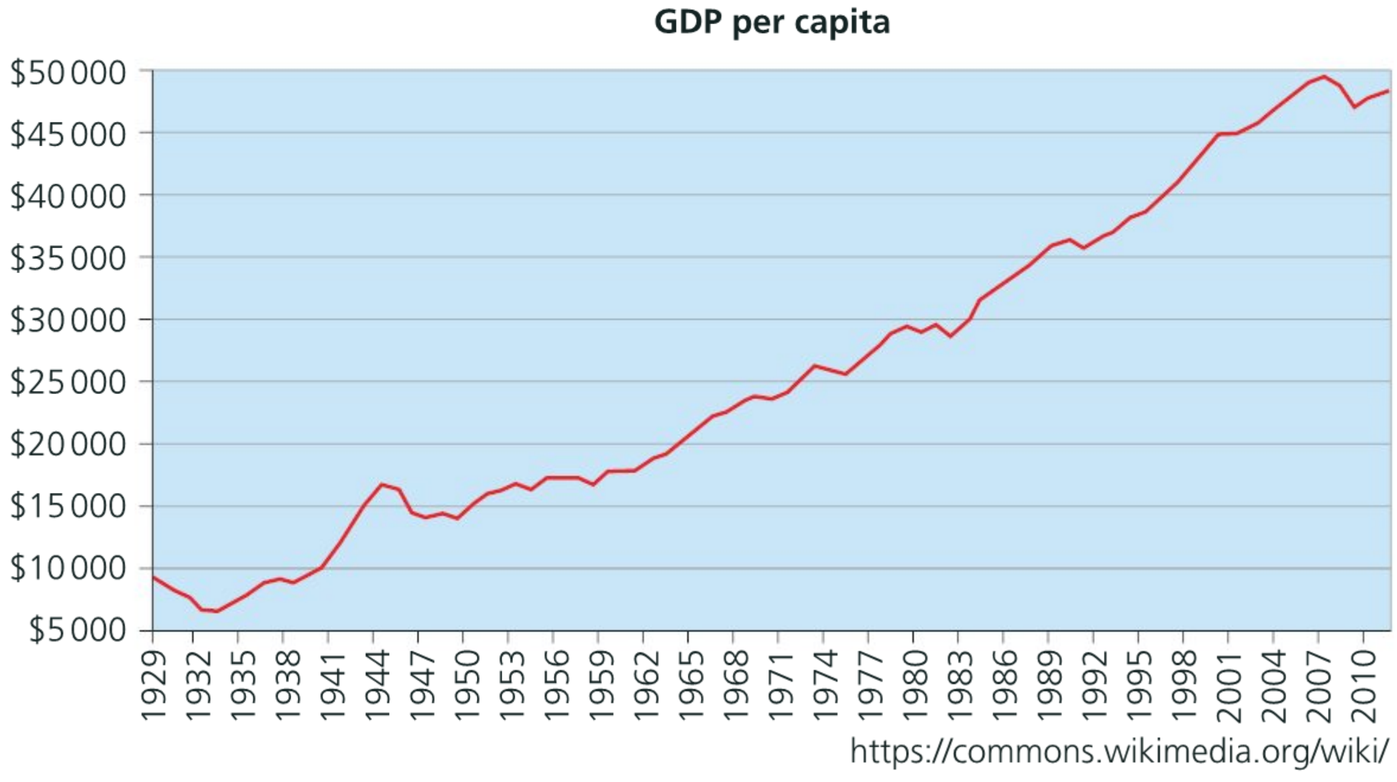


This bar graph indicates the 10 most spoken languages in the world. Do you notice an outlier? What might account for the reason this language is spoken so much more than the rest?



WHAT MAKES YOU SAY THAT?

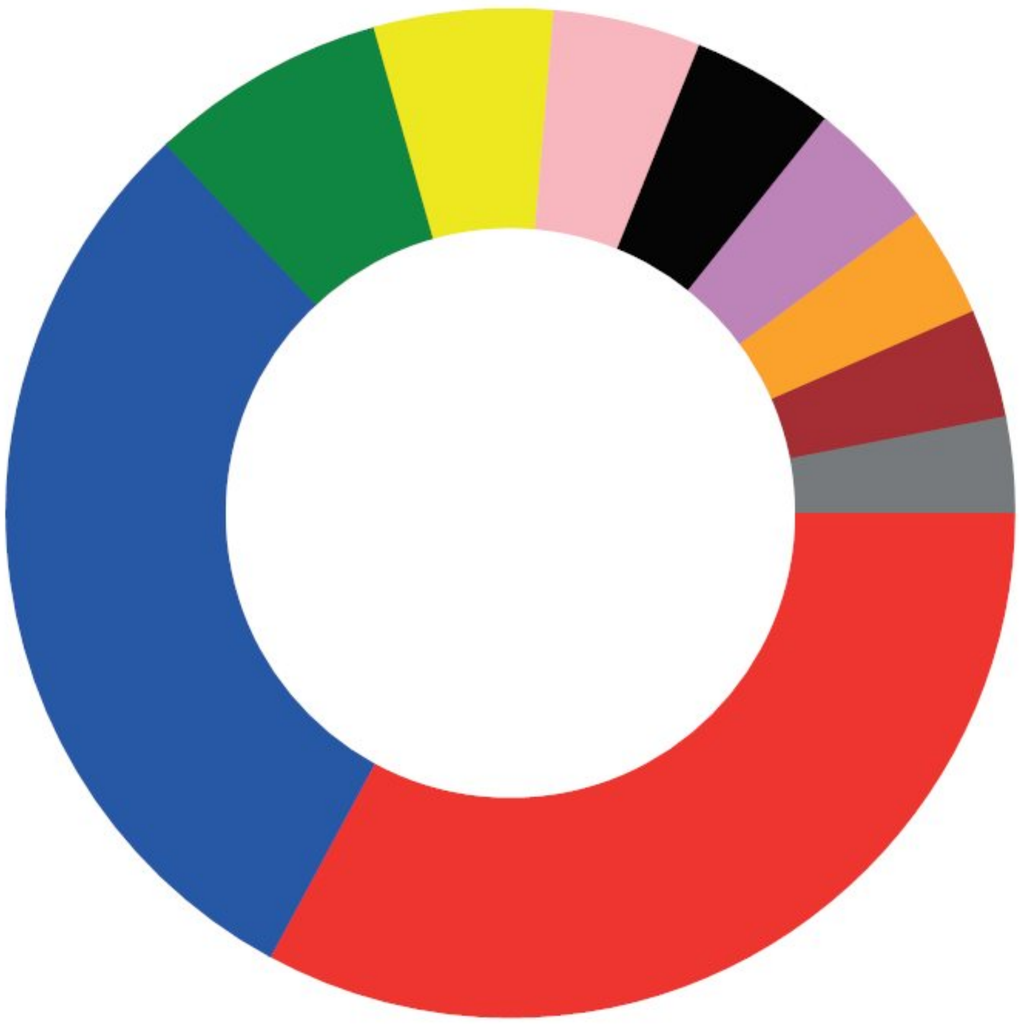
What story can this line graph tell us?



https://commons.wikimedia.org/wiki/

Can you spot the outliers? There are obvious peaks and valleys (or 'dips') in the graph. Look closely at the years when these occur. Did anything happen in the early 1930s that may have caused the world economy to dip? What major world event happened in 1945? How might this have affected the economy?

The 10 most populated countries in the world



Source: http://codecanyon.net/item/jquery-graphs-plugin/full_screen_preview/5957381

10 most populated countries

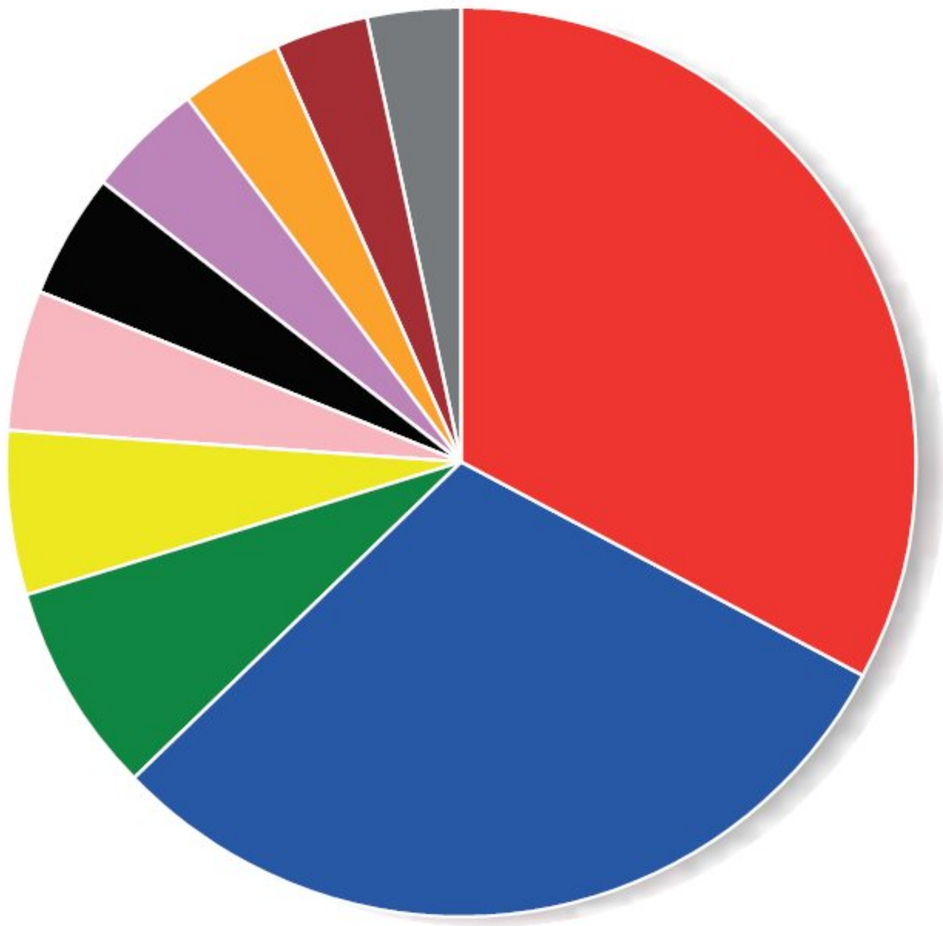
Colour	Country	Millions of people	Percentage
	China	1360	32.95%
	India	1235	29.92%
	USA	316	7.66%
	Indonesia	237	5.74%
	Brazil	201	4.87%
	Pakistan	184	4.46%
	Nigeria	173	4.19%
	Bangladesh	152	3.68%
	Russia	143	3.46%
	Japan	127	3.08%

In general, graphs are traditional, time-tested displays of information which, when used correctly, are very effective in communicating and analyzing data. Many more creative and customized graph types have evolved. Some are static (a quick picture) and others are dynamic (animated). Some are effective, and others are creative for the sake of being creative. Many are simply more attractive versions of a traditional graph!

One type of graph that has gained a lot of popularity is the doughnut chart. This is, essentially, a pie chart with the middle removed. Above is a doughnut graph of the 10 most populated countries in the world.

THINK-PAIR-SHARE

Here is a pie chart displaying the same information as the doughnut chart above.



What are some advantages and disadvantages of the doughnut chart?

Which chart do you think is more effective in displaying accurate, easy-to-read data?

What improvements could you make to either graph?

SUMMATIVE ASSESSMENT: Prove your conjecture!

ATL

- Information literacy skills: Finding, interpreting, judging and creating information
- Media literacy skills: Interacting with media to use and create ideas and information

A **conjecture** is an assumption that has little or no basis. Using data and statistics, you can turn your conjecture into a fact or myth – depending on your findings.

Your task is to come up with a real-life conjecture about whether there is a relationship between two variables. Does one affect the other? Then, gather primary data and **draw** a scatter plot to **prove** or disprove your conjecture. Some possible examples include:

- **Conjecture: the length of your legs affects how fast you can run. You could prove or disprove this by gathering primary data from classmates' leg lengths and the time it takes them to run 100 m.**
- **Conjecture: the amount of trash in each classroom depends on the number of students in its homeroom class. You could investigate this by surveying all the trash cans after homeroom/ at the end of the day and asking teachers for relevant class sizes.**
- **Conjecture: the hours spent at work by students' parents or guardians affect the number of extra-curricular activities they are involved in. You could prove or disprove this by conducting a survey.**

Your final assessment should include:

- **your conjecture as a title**
- **raw data – the information collected in table form**
- **a labelled scatter plot of your data**
- **an analysis of your scatter plot – be sure to include discussion of any trends, outliers, correlation**
- **a reflection – do your findings possess any ethical, cultural or environmental implications? What could you do to make your research more accurate?**

Assessment opportunities

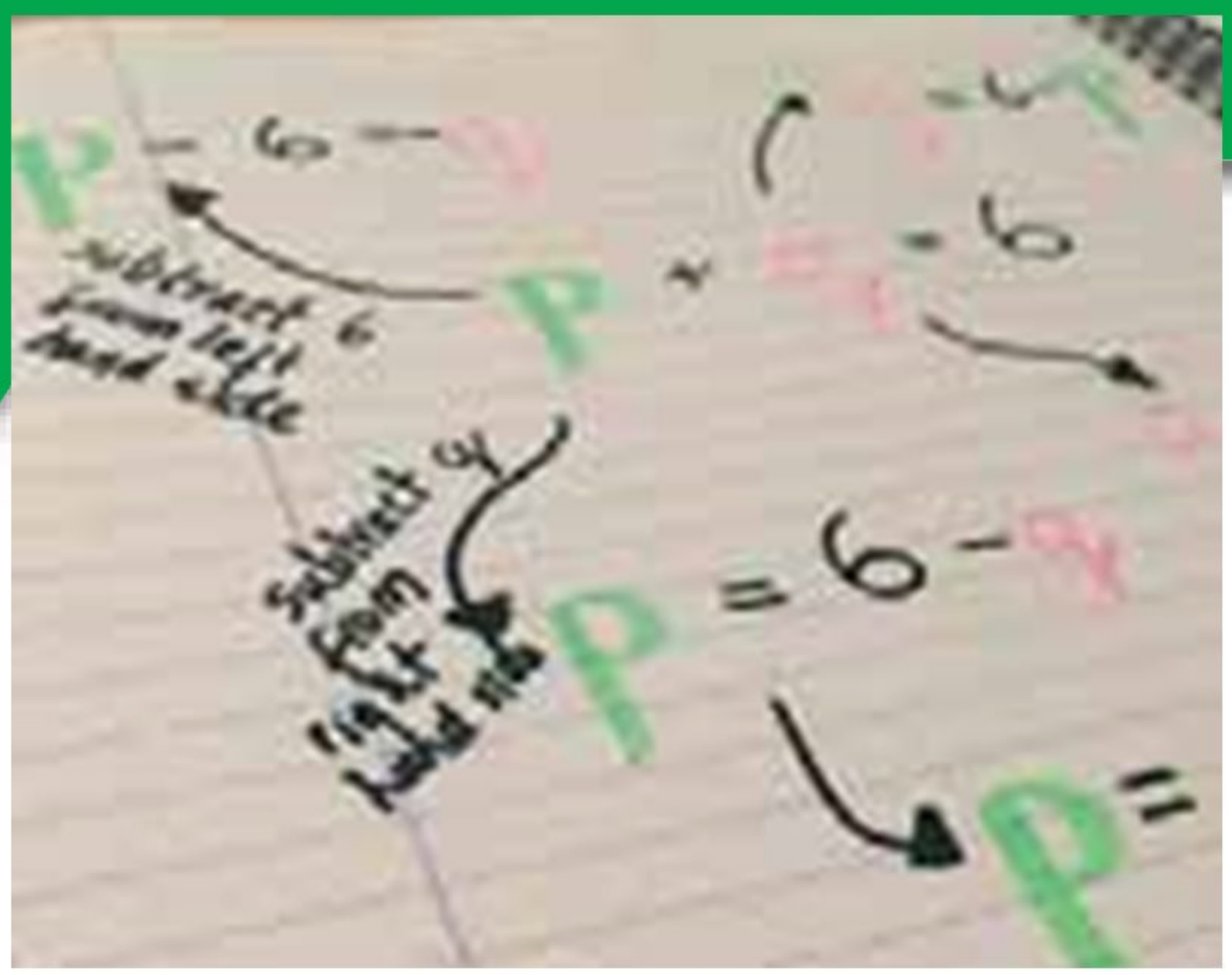
- This activity can be assessed using Criterion D: Applying mathematics in real-life contexts.

Reflection

Use this table to reflect on your own learning in this chapter.						
Questions we asked	Answers we found	Any further questions now?				
Factual: Where does data come from? What is a variable?						
Conceptual: Who needs data? Are data just a list of numbers? What sorts of stories can the data tell us?						
Debatable: Can everyone understand data? Can data provide answers to global problems? What kind of difference can one person make? Can graphs lie?						
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?				
		Novice	Learner	Practitioner	Expert	
Communication skills						
Information literacy skills						
Reflection skills						
Learner profile attribute(s)	How did you demonstrate your knowledge and reflection skills in this chapter?					
Knowledgeable						
Reflective						

4 Should we cross the bridge or keep everything in balance?

Unknowns and **variables** can be **modelled** and solved using algebraic **logic**, which can be **expressed** in different **personal and cultural** ways.



CONSIDER THESE QUESTIONS:

Factual: What does commutative mean? How can we change equations with multiplication and division to solve them? What happens when there is more than one operation to 'undo' to solve the equation?

Conceptual: What is the logic of the algebraic movements? Why is it important to define variables properly? How do we substitute into expressions or equations? Why do we substitute values into expressions?

Debatable: How true to real-life are these scenarios for solving and substituting? Are these methods better than or more complex than using common sense to solve a number problem? Can personal and cultural differences affect how we see algebra?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- the vocabulary needed to talk about algebra
- how to collect like terms
- how to multiply or divide terms with letters and numbers
- the correct communication for terms with different unknowns
- the order of operations for carrying out calculations.

IN THIS CHAPTER, WE WILL …

- **Find out** how substitution helps us to problem solve and prove our answers.
- **Explore** how algebra can be hidden in everyday situations.
- **Take action** by proving how online 'mystical mind-reading' tricks work using mathematical logic.

■ These Approaches to Learning (ATL) skills will be useful …

- Self-management skills
- Communication skills
- Information literacy skills
- Transfer skills

● We will reflect on these learner profile attributes …

- **Thinker** – using critical-thinking skills to analyze and take responsible action on complex problems; exercising initiative in making reasoned, ethical decisions.
- **Balanced** – understand the importance of balancing different aspects of our lives – intellectual, physical and emotional – to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

◆ Assessment opportunities in this chapter:

- **Criterion A**: Knowing and understanding
- **Criterion B**: Investigating patterns
- **Criterion C**: Communicating
- **Criterion D**: Applying mathematics in real-life contexts

KEY WORDS

equation equivalent

DISCUSS



How many 'like terms', or bananas, would you **estimate** that the street trader has on their bike? Compare your answer with a partner. How could you check to see who was right? What does this have to do with algebra?

Reflection

How much of what you learned in Chapter 2 could you recall quickly? What does that tell you about your personal learning strategies? How can you improve or extend them? What is the difference between short-term memory and long-term understanding?

WHAT ARE THE BASIC BUILDING BLOCKS OF ALGEBRA?

Remember from Chapter 2 that communicating in algebra follows certain rules or conventions. You may want to revisit what you learned in Chapter 2 before continuing with this chapter.

What does commutative mean?

WHAT IS COMMUTATIVITY?

The commutative property refers to the ability to move stuff around. You know that 3 and 5 when added together give a **sum** of 8. We can write this in the following way:

$3 + 5 = 8$

And you also know that

$5 + 3 = 8$

which tells you that addition is commutative.

In our example here, it doesn't matter if you add 3 to 5 or add 5 to 3. Either way will get you the right answer.

Can you think of another commutative operation? Can you remember a non-commutative operation, where the order **does** matter?

The two equations $5 + 3 = 8$ and $3 + 5 = 8$ are not the only possible versions that you know! It is also true that:

$3 = 8 - 5$ $\quad 5 = 8 - 3$ $\quad 8 = 3 + 5$ $\quad 8 = 5 + 3$

Can you think of any other possible combinations of this sum that are not listed here?

Hint

You can use negative numbers (or integers) in equations.

We can extend what we know instinctively about $3 + 5 = 8$ to situations with unknown numbers. So, if we know that two unknowns add to a constant like 6, we can write an equation to show the relationship. We can call the two unknowns anything we like, and for example:

$+ = 6$

Let's colour-code them so: can be green and can be pink. The signs and constant (6) will stay in black. If we start with the version of the equation above, we can deduce other versions of the equation which are also true, just as with the example of 3 and 5 equals 8 above.

What happens if we 'move' the around by subtracting from both sides of the equation? We could find easily if we knew the value for the unknown .

$+ - 6 = 0$

subtract 6 from both sides of the equation

$+ = 6$

subtract from both sides of the equation

$= 6 -$

'Moving' both the -term and the -term in this way gives us different versions of the same equation! There are more versions.

$- 6 = -$ $\qquad$ $= 6 -$

subtract 6 and subtract from both sides of the equation

subtract from both sides of the equation

$+ = 6$

subtract 6 and subtract from both sides of the equation

subtract from both sides of the equation

$- 6 = -$ $\qquad$ $= 6 -$

Here are even more versions of the same equation. Where they have come from? What rules can you see?

$- = - 6$ $\qquad$ $- = 6 =$

You have probably used this type of logic before, without realising that it was a form of algebra. Let's look at some examples of word problems that can be represented **algebraically**.

Examples

1 If I have two natural numbers, one of which is 7, and their total is 11, what is the other number?

Solution

Did you know instantly that the second number must be 4? This can be shown as:

$x + 7 = 11$

We can find the other number, now called x, by subtracting the 7:

$x = 11 - 7$

Which gives the answer:

$x = 4$

2 If two teams are playing a hockey game and the score is 5 goals to 2, what is their goal difference? Write two equations to describe this situation and give the answer for the goal difference.

Solution

$2 + \quad = 5$ $\qquad\qquad$ $= 5 - 2$

The goal difference is 3.

3 There are twice as many students in a Grade 6 class at Branksome Hall who prefer to watch movies on their laptop as those who watch on a TV. If seven students prefer to watch movies on a TV screen, how many students watch on their laptops? Use an equation to support your response.

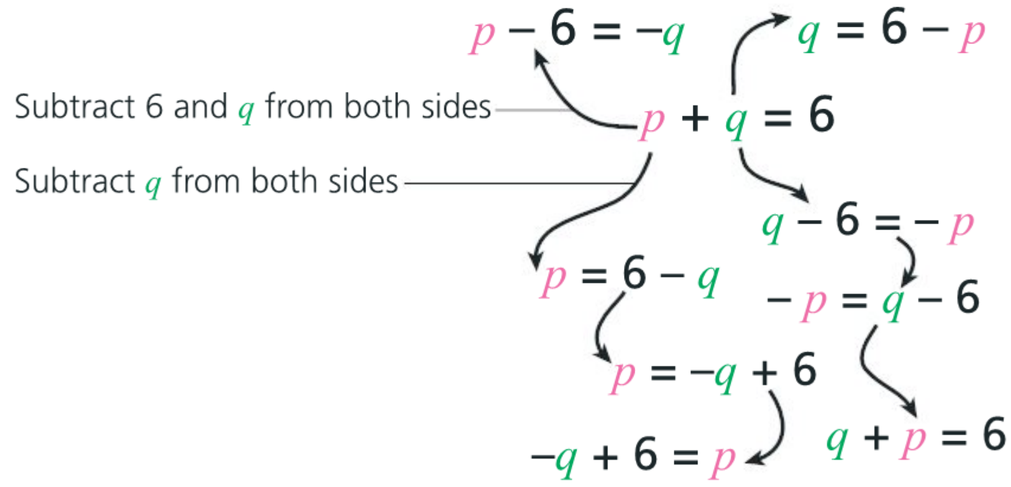
4 How many students are there in total in the example above? How many **more** prefer to watch on laptops than on TV? Use an equation to support your response.

ACTIVITY: Describe the move

ATL

- Communication skills: Make inferences and draw conclusions

Here are different values for the equation $\quad + \quad = 6$.



The 'move' or algebraic operation has been explained along two of the arrows. You can see a description of what is happening mathematically to move from one equation to another (equivalent) version. This description explains the logical change or operation that took place.

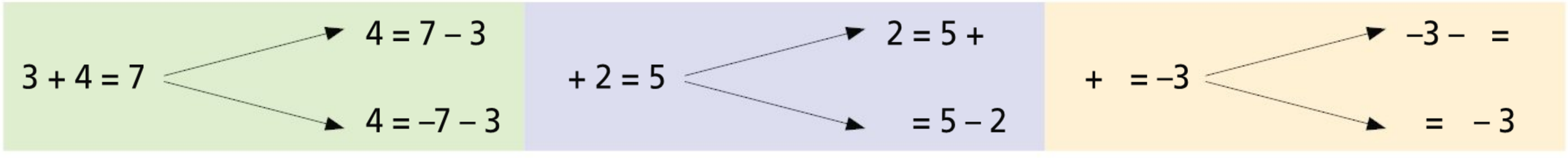
For each of the other arrows provided, describe what is happening; identify the moves and operations that are taking place to make the new equations. You can add more arrows, and more equations, to extend your knowledge!

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding and Criterion C: Communicating.

PRACTICE EXERCISE

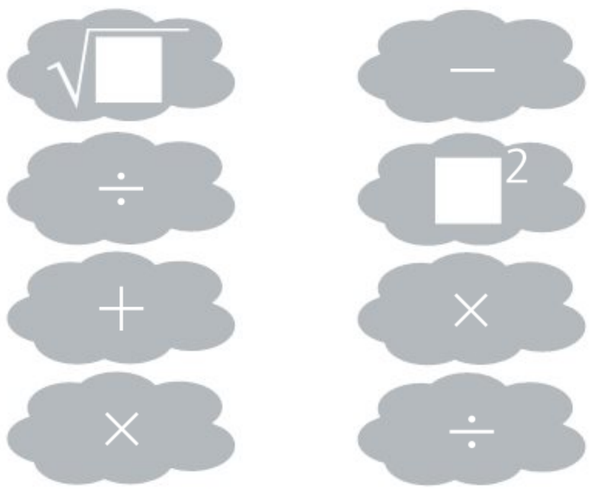
Which of the two variants given in each case is not a correct rearrangement of the initial equation?



What is the logic of the algebraic movements?

What makes you say that?

Match the operation with its **inverse** by drawing lines between each operation and the exact opposite operation (the inverse).



Now that we know a single equation can be written in many different ways, let's take a closer look at how we can use these forms. We will look at these rearrangements in two different ways, both of which can help us when solving such equations.

METHOD 1: OVER THE BRIDGE

In this method, you see the expressions in an equation as operations that have to be **undone** or **reversed**. The reversing operation is called **inversing**. Sometimes the equals sign is viewed as a portal or bridge – when you cross it you 'do the opposite' and examples sometimes show arrows travelling across the equals. You can even imagine that the operation has 'gone to the dark side' and is the opposite of what it was before!

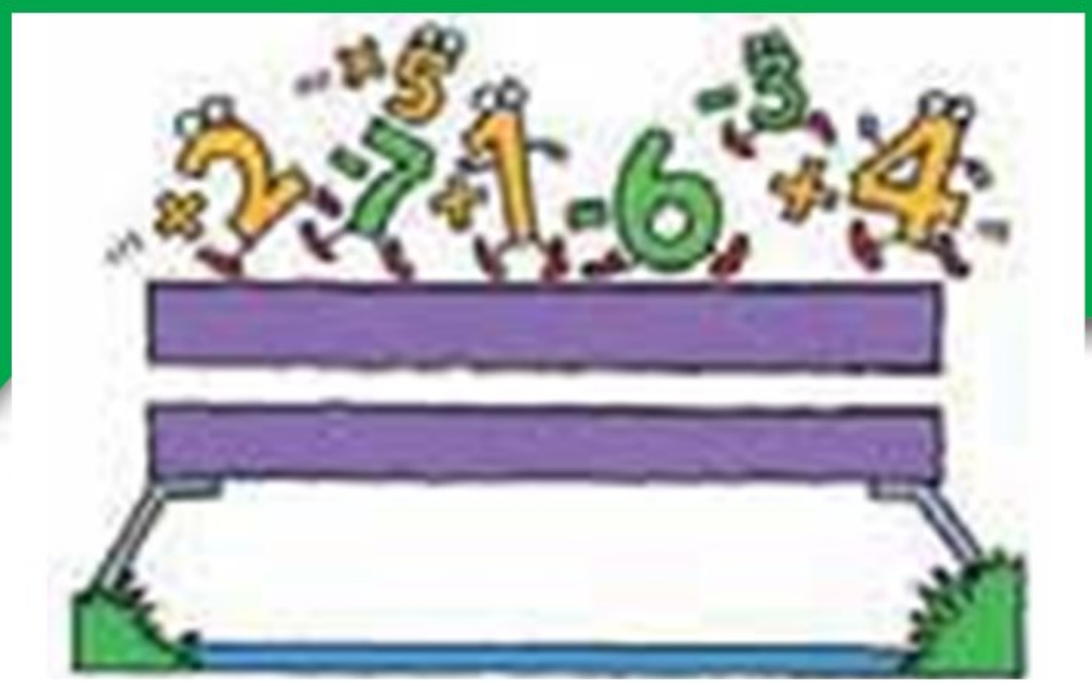
Example 1

Let's look at this problem:

$$x + 5 = 10$$

We know that a number has been added to 5 to give the answer of 10.

Was it obvious that the opposite of adding 5 is to subtract 5 from 10? Use your instinct every time you want to reverse an operation to find an unknown.



$x + 5 = 10$

$x + 5 = 10$ (–5 is the inverse of +5)

$x = 10 - 5$

$x = 5$

So we know the unknown number, x, was 5. We have used good clear lines of communication to show what we have done to solve the problem too.

Example 2

$3x = 12$

$3x = 12$ (multiply × 3 becomes ÷ 3 on the other side)

$x = 4$

Make sure you show the inverse operation on the right-hand side and **state** your final answer clearly, using an equals sign.

Example 3

$-0.5 = 6$

$= 6 + 0.5$

$= 6.5$

PRACTICE EXERCISE

Using method 1, solve:

1 $a + 2 = 8$ **3** $c - 3 = 10$ **5** $5 + e = 9$

2 $\quad + 1 = 0$ **4** $\quad - 4 = -2$ **6** $3 - \quad = 3$

Reflection

What do you think of this method? Can you explain it in your own words? What appears good or bad to you about this method?

METHOD 2: BALANCING

The other common method is to view the equation as a balance that must be maintained at all times – what you do to one side, you must also do to the other. You wish to simplify the relationship, not to change it fundamentally and put it out of balance.

Example 1

$x + 5 - 5 = 10 - 5$

$x + 0 = 5$

$x = 5$

Example 2

$3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

In this method, every step towards **isolating** the singular variable is a balanced operation on each side; students often show the operation they are doing under both sides to explain their method. You will grow more confident and your lines of reasoning will become more concise.

Hint

Some people like to use arrows to show what they have done or write the operation on the side to help with communication. As you become more confident this may not be necessary, but at first it might be a helpful support.



Example 3

$-0.5 = 6$

$+0.5 \quad +0.5$

$= 6.5$

PRACTICE EXERCISE

Using method 2, solve:

1	$a + 2 = 8$	**3**	$c - 3 = 10$	**5**	$5 + e = 9$
2	$\square + 1 = 0$	**4**	$\square - 4 = -2$	**6**	$3 - \square = 3$

Reflection

What do you think of this method? Can you explain it in your own words? What appears good or bad to you about this method?

Use whichever representation (method) you prefer to solve the following problems.

PRACTICE EXERCISE

Solve:

1 $x - 3 = 5$

2 $\square - 9 = 1$

3 $\square - 16 = 11$

4 $\square + 20 = 45$

5 $\square + 15 = 12$

6 $7 + \square = 11$

7 $7 - \square = 11$

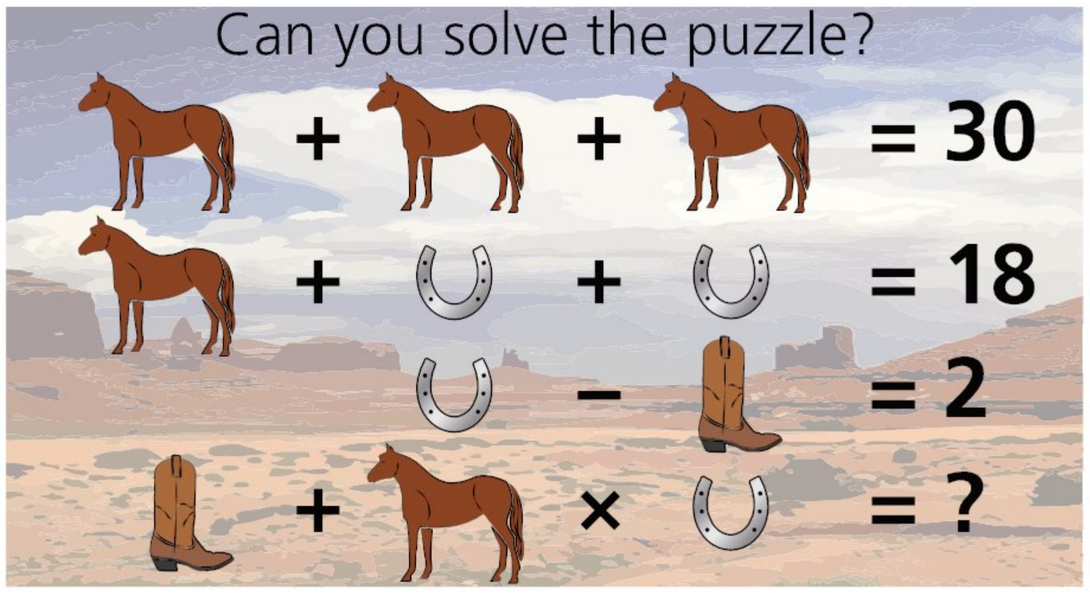
8 $3.5 + \square = 9$

9 $3.5 - \square = -9$

How can we change equations with multiplication and division to solve them?

Another pair of inverse operations is multiplication and division.

Let's use algebra to **solve** the horse problem shown here.



How can you tell a horse is worth 10 points?

$\quad + \quad + \quad = 30$

Therefore

$\quad 3 \quad = 30$

So

$\quad = \frac{30}{3} = 10$

We have proved that a horse is worth 10 points.

Spoiler alert! We will keep working on this puzzle in later pages, but feel free to go ahead and solve it now, if you can see a good strategy to find the unknowns.

This method can be used to calculate the cost of individual items when you know the total price.

DISCUSS

What is the largest number that can be created when a two-digit number is **decreased** by the **product** of its digits?

Hint

As you can see in the question above, not all answers are integers (positive or negative whole numbers). For example $8x = 20$ can be solved by dividing both sides by 8.

This gives the answer $8x = \frac{20}{8}$ before you simplify it,

or

$x = \frac{5}{2}$ in the form of an **improper** (or 'top heavy' fraction)

or

$x = 2\frac{1}{2}$ when written as a **mixed number**

or

$x = 2.5$ when written as a decimal.

All of these answers are correct and equivalent. Check the question to see which of the forms is required. If the question doesn't say, you can give any of the forms – except the unsimplified fraction. Leaving your answer like that is not completely correct and would amount to poor communication.

Examples

1 A box of 12 lightbulbs costs 36 euro. How much does one cost?

Solution

Let represent the cost of one lightbulb.

$12 \quad = 36$

so

$\quad = 3$

2 The school talent show lasts an hour and a half and has 12 acts. How long does each act get?

Solution

12 acts must be shared into 90 minutes.

$12 \quad = 90$

$\quad = \frac{90}{12}$

$\quad = 7.5$ minutes each

3 If it costs $130 to rent a party venue for four hours, how much does it cost for a single hour?

Solution

Let represent the cost of one hour.

$4 \quad = 130$

$\quad = \frac{130}{4}$

$\quad = 32.5$

The venue costs $32.50 per hour.

WHAT IF THE NUMBER HAS BEEN DIVIDED INSTEAD OF MULTIPLIED?

On a snowy day, one-fifth of the students in Grade 6 can't get out of their homes, which results in nine students being absent. How many students are there in the whole of Grade 6?

Let represent the number of students in Grade 6:

$\frac{}{5} = 9$

We must multiply both sides by 5 to find :

$= 9 \times 5$

$= 45$ students

Here is another example:

$\frac{x}{7} = -11$

Multiply both sides by 7:

$x = -77$

PRACTICE EXERCISE

Solve:

1	$4x = 12$	**4**	$5 = 20$	**7**	$12 = 240$
2	$10x = -200$	**5**	$5 = 200$	**8**	$3 = 693$
3	$16x = 21$	**6**	$9 = -81$	**9**	$6 = 720$
10	$9 = 12$	**13**	$11 = 41$	**16**	$13 = 20$
11	$7 = 20$	**14**	$2 = 411$	**17**	$15 = 70$
12	$16 = 8$	**15**	$12 = -9$	**18**	$6 = 8.5$

19 A number divided by three is equal to five. What was the original number?

20 $\frac{}{3} = 5$

21 $\frac{}{10} = 4$

22 $\frac{}{6} = 2$

23 $\frac{}{6} = -2$

24 $\frac{}{10} = 2.5$

25 $\frac{}{3} = -\frac{1}{3}$

26 The number of students in a school has increased by a tenth this year, which means there are 50 new pupils. How many students

a were at the school last year

b are at the school this year?

ACTIVITY: Algebra in the real world

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations

For each of the three scenarios, A to C:

1. **identify the relevant mathematical elements to create an equation**
2. **solve the equation**
3. **comment on how accurate or true to real life your answer is. Do you think any of these answers might change in real life for any reason? Why or why not?**

Scenario A: a basket of fruit

A trader is cycling with a basket of 28 pineapples on the front of her bicycle weighing 14 kg. How much does each pineapple weigh?

Scenario B: digital photos in a folder

A folder on your computer contains 300 photographs and is taking up 15 000 MB (megabytes) of space. How much space does each photograph use up?

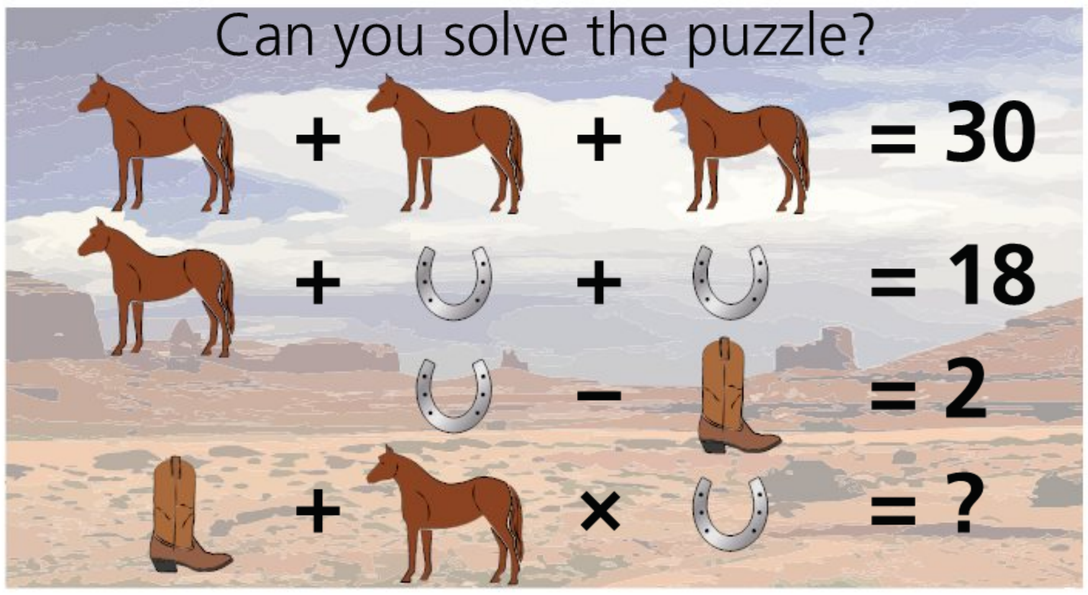
Scenario C: shelving

A frame is 200 cm high and comes with 11 shelves. You want to place the shelves at least 15 cm apart. How many shelves will you use from the 11 provided?

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

What happens when there is more than one operation to 'undo' to solve the equation?



Let's return to our puzzle from earlier. We already know that the horse is worth 10 points.

If we look at the second equation:

horse + horseshoe + horseshoe = 18

This can be written as an equation:

10 + 2 = 18

Remove the 10 from the both sides:

2 = 8

$\frac{2}{} = \frac{8}{2}$

Two horseshoes are worth 8 points, so one horseshoe is worth 4 points:

= 4

TWO-STEP SOLVING OF EQUATIONS

In the puzzle we have been working on, there were two operations that had to be undone to figure out what the horseshoes were worth. To find the value for one horseshoe, first we had to subtract 10 and then divide the answer by two.

It is very important to identify which operation to do first. **Always undo addition or subtraction before multiplication or division.** This **undoing** takes the **reverse** order of operations compared to those you learned before. (Remember your acronym from Chapter 1? Was it BEDMAS, or something else?) Think of this undoing as retracing your steps through your acronym.

Example 1

$$5x - 2 = 23$$

Isolate the x term first by moving –2:

$$5x - 2 = 23 + 2$$

$$5x = 25$$

Now find $1x$ by dividing the right-hand side by 5:

$$5x = 25 \quad \div 5$$

$$x = 5$$

Example 2

$$\frac{x}{4} + 2 = 5$$

Isolate the x term by moving +2 over:

$$\frac{x}{4} + 2 = 5 - 2$$

$$\frac{x}{4} = 3$$

Now move the ÷4 to the other side:

$$\frac{x}{4} = 3 \quad \times 4$$

$$x = 12$$

Example 3

$$\frac{18x + 3}{6} = 3.5$$

As the entire left-hand side is a fraction, move the ÷6 first:

$$\frac{18x + 3}{6} = 3.5$$

$\times 6$

$$18x + 3 = 21$$

-3

$$18x = 18$$

$$x = 1$$

And now we can look at the same two-step questions done using the balancing method.

Example 1

$$5x - 2 = 23$$

$+2 \quad +2$

$$\frac{5x}{5} = \frac{25}{5}$$

$$x = 5$$

Example 2

$$\frac{x}{4} + 2 = 5$$

$-2 \quad -2$

$$\frac{x}{4} = 3$$

$\times 4 \quad \times 4$

$$x = 12$$

Example 3

$$\frac{18x + 3}{6} = 3.5$$

$\times 6 \quad \times 6$

$$18x + 3 = 21$$

$-3 \quad -3$

$$\frac{18x}{18} = \frac{18}{18}$$

$$x = 1$$

How do you decide which step to do first? Remember that we are trying to isolate the variable.

Explain how you decide what to inverse or 'undo' first, while remembering the **order of operations** rules.

PRACTICE EXERCISE

Solve:

1	$2x + 1 = 7$	**5**	$10x + 5 = 75$
2	$2x + 1 = 11$	**6**	$8x + 2 = 34$
3	$3x + 1 = 13$	**7**	$9x + 17 = 35$
4	$3x + 1 = 61$	**8**	$40x + 60 = 860$
9	$2x - 1 = 8$	**13**	$12x - 9 = 51$
10	$3x - 3 = 24$	**14**	$19x - 2 = 17$
11	$6x - 9 = 15$	**15**	$21x + 19 = 40$
12	$7x - 3 = 74$	**16**	$3x - 3 = -24$

17 $7x - 3 = -10$

18 $5x + 5 = -200$

19 $5x - 5 = -200$

20 $2.5x + 1 = 6$

21 $2.5x + 1 = 3.5$

22 $19x + 1.9 = 58.9$

23 $\frac{x}{7} + 1 = 7$

24 $\frac{x}{5} - 20 = 10$

25 $\frac{x}{9} = 7$

26 $\frac{3x}{9} = 7$

27 $\frac{2x}{9} = 7$

28 $4x + \frac{2}{3} = 6$

Why is it important to define variables properly?

Consider solving a more complicated equation, such as:

$2x^2 - 2 = 240$	$2x^2 - 2 = 240$
$2x^2 - 2 = 240$ $\quad +2$	$2x^2 = 242 \quad \vert +2$
$2x^2 = 242$ $\quad \div 2$	$x^2 = 121 \quad \vert \div 2$
$x^2 = 121$ $\quad \sqrt{}$	$x = \sqrt{121} \quad \vert \sqrt{}$
$x = \sqrt{121}$	$x = 11$
$x = 11$	

We will get the same answer with both processes, as long as we move or inverse the operations in the correct order.

Many people with messy bedrooms, lockers or desks insist they know where everything is, but no one else can find things in the mess. Similarly, with algebra, you may know what your variables mean and where your equation and final answer came from, but nobody else will understand your logic unless you lay it out clearly and neatly.

PRACTICE ACTIVITY

Word problems



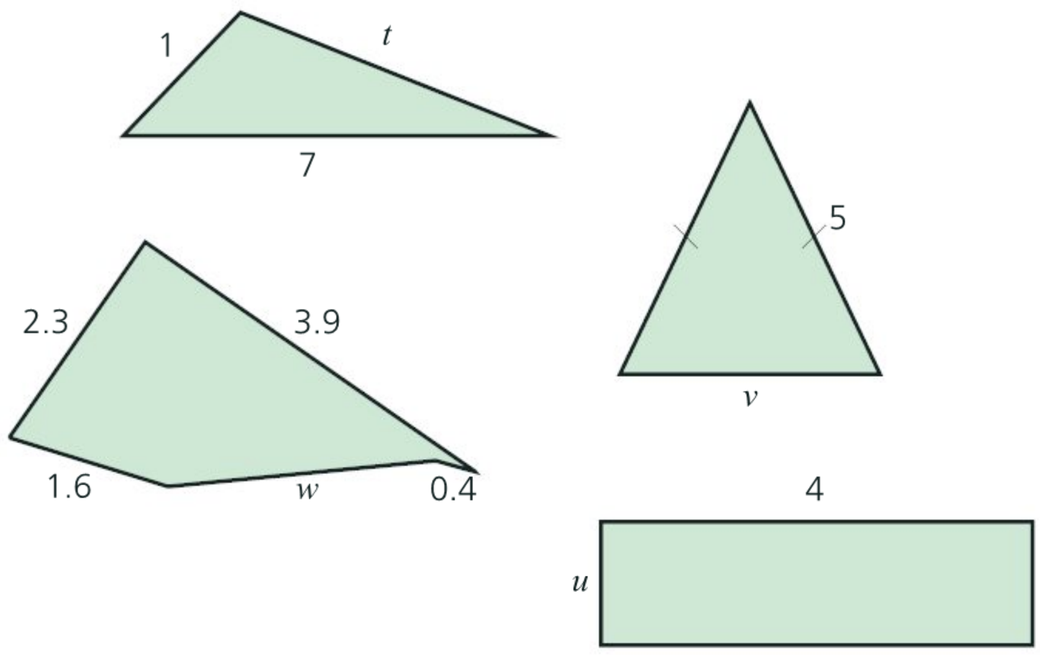
1 Jake is 180 cm tall and the height difference between him and his younger brother Alvin is 10 cm.

- **a Write an equation to show this relation.**
- **b Solve to find Alvin's height.**
- **c Are there any other possible solutions to this scenario? Why or why not?**

2 Two consecutive houses have numbers which make a sum of 101. What are the house numbers? Show your working.



3 The perimeter of a shape is the total length of the sides of a shape. For all the following images, the perimeter is 11. Use algebra to find the missing sides. Show your working.



4 If 120, divided by a certain number minus 2, is equal to 10. What was the original number? Trish says the number is 14 and Imran says the number is 10.

- **a Who is correct? How did you work it out?**
- **b Can you work out what the mistake was for the person who was wrong? Write down what they did, in algebraic or mathematical terms to show their (incorrect) reasoning.**

ACTIVITY: The Wild Wadi waiting game

ATL

- Communication skills: Reading, writing and using language to gather and communicate information; Make inferences and draw conclusions



Fiachra and Sarah are excited to visit the Wild Wadi Water Park and want to calculate how many rides they can go on. They estimate that each ride will last around 7 minutes, including walking, waiting, climbing and being on the ride itself.

Let x represent the number of rides.

1 **Write an expression for the total time it will take to go on the rides.**

They also know that it takes 5 minutes to go from the entrance to the first ride and 5 minutes to get out of the park again at the end.

2 **Write a new expression for the total time it will take to visit the park (including taking rides and walking to and from the entrance).**

The park is open for 6 hours, or 360 minutes, and they plan to be there for the whole day.

3 **Write an equation to show the time spent at the water park.**

4 **Solve the equation to determine . State how many rides they will go on.**

When they get there, the water park is really busy. Each ride actually takes 20 minutes because of the extra waiting.

5 **Write a new equation to show the time spent at the water park.**

6 **Solve the equation to determine . State how many rides they will actually go on.**

7 **How accurate or realistic do you think this answer is?**

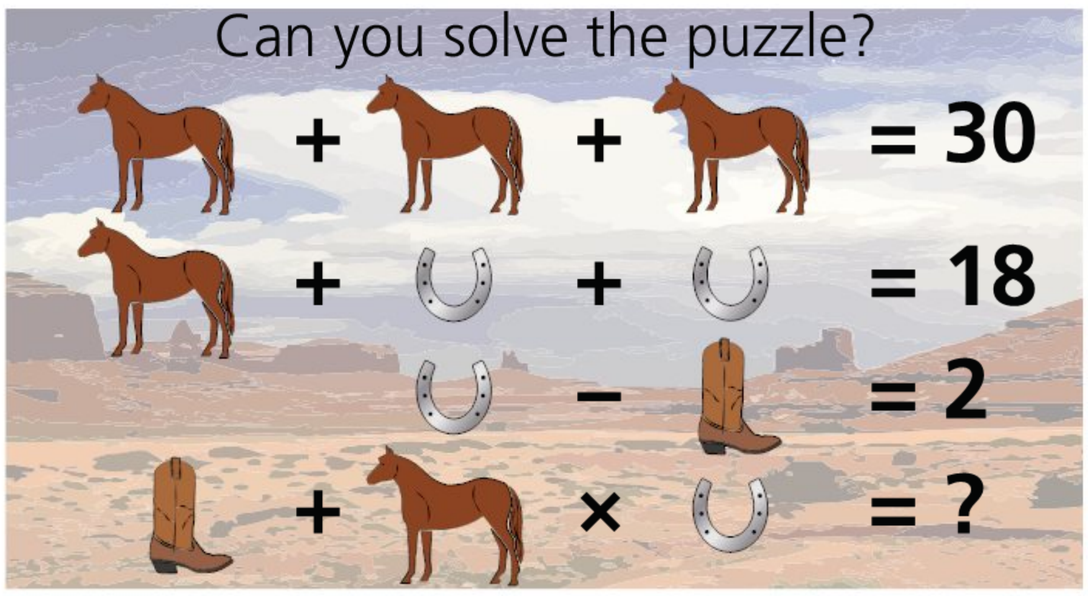
Assessment opportunities

In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

How do we substitute into expressions or equations?

SUBSTITUTION

The act of replacing one thing with another is called substitution. If we substitute a player in a sport, they take the place and role of a previous player. In algebra, if we input a value for a variable, or give a known number for an unknown, this is **substitution**.



Earlier, we solved the first equation to find that a horse was worth 10 points. Later, we rearranged an equation to find out that a horseshoe was worth 4 points. Next we are going to look at the third equation which tells us:

horseshoe – boot = 2

Now we **substitute** the value we know for the horseshoe into the equation to get:

4 – boot = 2

So, the boot must be worth 2 points.

THINK-PAIR-SHARE

Recreate the algebra puzzle but for different values. Exchange your new puzzle with a classmate and **solve** each other's puzzle. (You can change the symbols if you like but make sure all the values are correct by substituting them into the equations before you swap.)

Imagine the unknown letter in an equation is a position in sport that can be replaced with a variety of different players. This term can be replaced, if you know a certain value for that letter. We call this substituting too.

For example, take this expression: 2 + 1

Now find the value of the expression, if:

= 2 = 11 = 0.5

2(2) + 1 = 5 2(11) + 1 = 23 2(0.5) + 1 = 2

Each of these different answers gives the correct value of the expression for a different value of .

Back to the puzzle … We still have one more equation to solve, but this time there are no unknowns. All we have to do is to substitute for all the terms we have solved.

boot + horse × horseshoe = ?

Substituting each of the values we have found gives us a much simpler expression:

2 + 10 × 4 = ?

Which operation must we do first? What is the final answer to the puzzle?

PRACTICE EXERCISE

Solve:

1 10 + 4 when

a = 1 **c** = 10

b = 3 **d** = –5

2 3a – 2 when

a a = 1 and = 2 **c** a = –1 and = 1

b a = 0 and = 3 **d** a = –2 and = –2

3 $12c^2$ when

a c = 0.5 **d** c = 0

b c = 1 **e** $c = \frac{1}{2}$

c c = 4 **f** c = – 5

Why do we substitute values into expressions?

Substitution gives you a value for an expression in a certain situation, given certain conditions. This can help solve problems, like the horse puzzle. Substitution is also a really helpful tool for checking your answers in problem solving.

Take your answer and substitute it into the original equation; if you come out with the same answer as the question, then you know you got the answer correct!

For example:

$$6 - 3 = 15$$
$$6 = 15 + 3$$
$$6 = 18$$
$$= \frac{18}{6}$$
$$= 3$$

Then substitute your answer, $= 3$, into $6 - 3 = 15$, the original question, to check:

$$6(3) - 3 = 18 - 3$$
$$= 15$$

Substitution can be used to check a complex example:

$$6 - x^2 = 2$$
$$-x^2 = 2 - 6 \quad \text{isolate the } x\text{-term}$$
$$-x^2 = -4$$
$$x^2 = 4 \quad \text{divide by } -1$$
$$x = \sqrt{4} \quad \text{inverse the square by using square root}$$
$$x = 2$$

We can now check by substituting this value into the original equation:

$$\text{If } \quad x = 2$$
$$6 - x^2 = 2$$
$$6 - (2)^2 = 2$$
$$6 - 4 = 2$$
$$2 = 2$$

Now you know the answer is correct!

Using the answers you found previously for these questions, check your answers are correct by substitution. (if you can't find your answers, solve them quickly again. It's good practice to repeat these types of skills to master them.)

PRACTICE EXERCISE

Solve:

1	$2x + 1 = 7$	**5**	$10x + 5 = 75$
2	$2x + 1 = 11$	**6**	$8x + 2 = 34$
3	$3x + 1 = 13$	**7**	$9x + 17 = 35$
4	$3x + 1 = 61$	**8**	$40x + 60 = 860$
9	$2x - 1 = 8$	**13**	$12x - 9 = 51$
10	$3x - 3 = 24$	**14**	$19x - 2 = 17$
11	$6x - 9 = 15$	**15**	$21x - 19 = 40$
12	$7x - 3 = 74$	**16**	$3x - 3 = -24$
17	$7x - 3 = -10$	**23**	$\frac{x}{7} + 1 = 7$
18	$5x + 5 = -200$	**24**	$\frac{x}{5} - 20 = 10$
19	$5x - 5 = -200$	**25**	$\frac{x}{9} = 7$
20	$2.5x + 1 = 6$	**26**	$\frac{3x}{9} = 7$
21	$2.5x + 1 = 3.5$	**27**	$\frac{2x}{9} = 7$
22	$19x + 1.9 = 58.9$	**28**	$4x + \frac{2}{3} = 6$

How true to real life are these scenarios for solving and substituting?



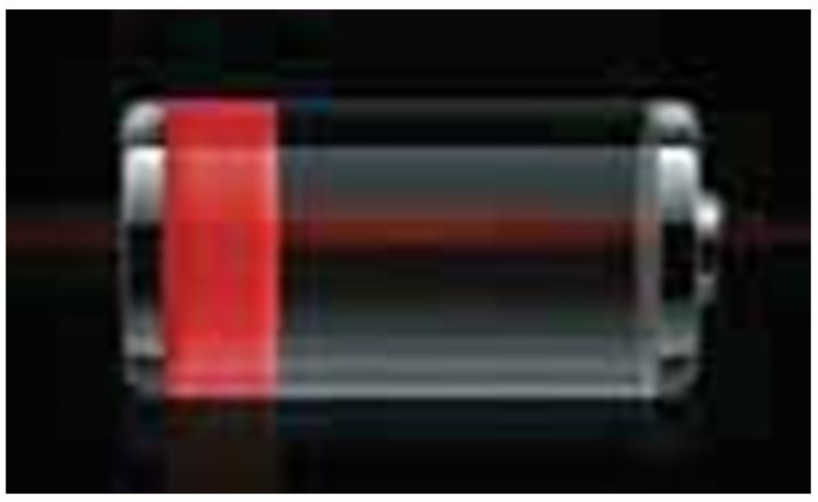
Let's consider this question by inquiring into some real-life situations.

ATL

- Communication skills: Understand and use mathematical notation

FAMILY MATTERS

Show your working for each of these questions. Remember to define your variables!



1. A phone indicates that 20% or one-fifth of battery life remains. This amount of charge lasts for another three hours. What is the battery life for a fully charged phone?
2. In a family of six children, each child is 2 years older than their next sibling. The oldest sibling is 18 years old.
 - **a** How old is the second oldest child?
 - **b** How old is the youngest child?
 - **c** What is the sum of the ages of all the children now?
 - **d** What will the sum of ages of all the children be in 10 years?
 - **e** What will the sum of the ages be in 20 years from now?
 - **f** Can you make a general rule that would predict their total age sum?

SPEED OF SOUND

Mach 1 is defined as the speed of sound. A speed can be given relative to, or as a multiple of, the speed of sound. For example, a jet flying at Mach 2 is travelling at two times (or twice) the speed of sound.

If we assume the speed of sound is approximately $340\,m\,s^{-1}$, how fast is:

1. Mach 1?
2. Mach 2?
3. Mach 3?
4. The X-15, at Mach 6.72, is one of the fastest manned aircraft. How fast can this aircraft travel in $m\,s^{-1}$?
5. The NASA X-43, at Mach 9.6, is one of the fastest aircraft. How fast can this aircraft travel in $m\,s^{-1}$?
6. If subsonic means slower than the speed of sound and supersonic means faster than the speed of sound, which of these speeds are subsonic and which are supersonic?

Speed	Supersonic or subsonic?
Mach 2.5	
Mach 0.5	
Space Shuttle Mach 1.1	
The speed of a cruise ship	

Links to: Sciences

Did you know that the end of a whip, the part that makes the cracking sound, moves faster than the speed of sound and creates a sonic boom? To see more about this phenomenon and other interesting things about Mach speeds, search for **sonic boom**, **sound barrier**.

SINGAPORE TRANSPORT SYSTEM



■ EZ-Link card used for transit payments in Singapore

The EZ-Link card is a contactless smart card that is used on the Singapore public transport network. An adult EZ-Link card costs S$12 (12 Singapore dollars), inclusive of a S$5 non-refundable card cost and a S$7 card value that can be spent on travel.

Neal purchased an EZ-link card and loaded some money onto the card for future travel. Let = amount of travel credit on the card and = total money spent by Neal.

1 Write an equation to **show** the relationship between the money spent and the travel credit.
2 If Neal spent 50 dollars, find how much credit he has on his card to travel.
3 If Neal spent 500 dollars, find .
4 If he has 275 dollars of credit on his card, how much did he spend?

MUSIC STREAMING

A streaming service allows for two user profiles on an account for £30 but then you must pay an extra £10 for any additional users you would like to add.

1 Write an expression and/or an equation to represent this information.
2 How much would you have to pay for:
 a a home with two people
 b a family of five people
 c a college dorm with 20 students
 d a rehabilitation centre with 35 patients
 e a military submarine with 104 personnel?

◆ Assessment opportunities

◆ In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

Can personal and cultural differences affect how we see algebra?

MEET A MATHEMATICIAN: VIETNAMESE TRADERS

Learner profile: Balanced



Whether they realize it or not, these traders on their bicycles are perfect examples of balance in action. These lifelong learners exhibit balance by understanding that if you add a load on one side of the bike, then you must add on the other side too. The same is true for subtracting from the load, and doubling it – whatever is done on the left must be done on the right.

Dutch Photographer, Loes Heerinck captured these images by waiting patiently on a bridge where she had a unique view. How is this different to the usual view we would have of such traders? What do we learn from this unique personal perspective? Search: **Loes Heerinck Vietnam** to view more of these beautifully mathematical, thoughtful and colourful images.

Sometimes we can see mathematics in the world around us, not only in numbers but in logic and form. If we consider the whole bike as an equation:

- how could the ideas of balance we have learned in this chapter be applied to the situation?
- how are the goods on the bike similar to the terms in an equation?
- how could we use the bike as a model for solving an equation?

When Loes heard that her photographs could be seen as mathematical in nature and were going to be used in this algebraic way, she said this:

> 'I find it very cool that my images could be used to explain that! Mathematics used to be my favourite subject!'

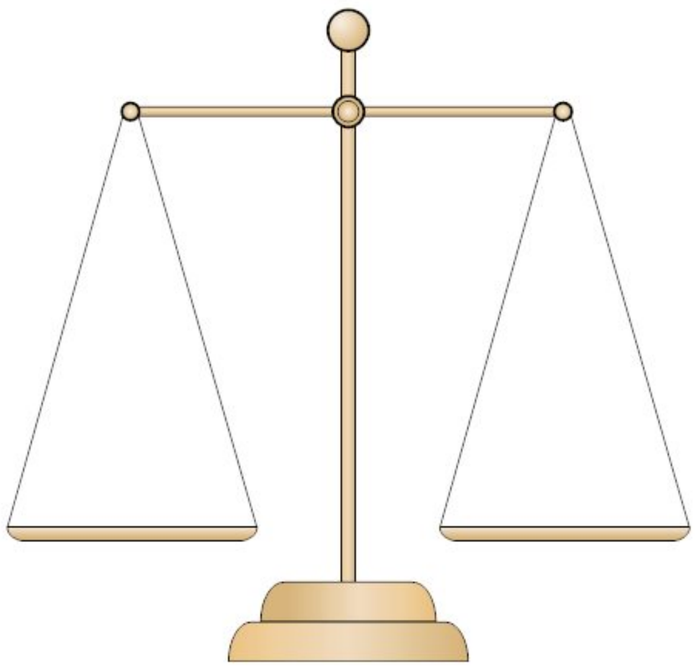
Links to: Sciences

The loads are not necessarily equal in **weight**, but the **forces** on the bike must remain in balance. Remember that the road and the rider also exert forces on the bike to maintain the **equilibrium**. If a rider takes away part of the load from the left-hand side, then they must also take away a load with an equal force from the right-hand side, or the bike will become imbalanced and the goods will fall.

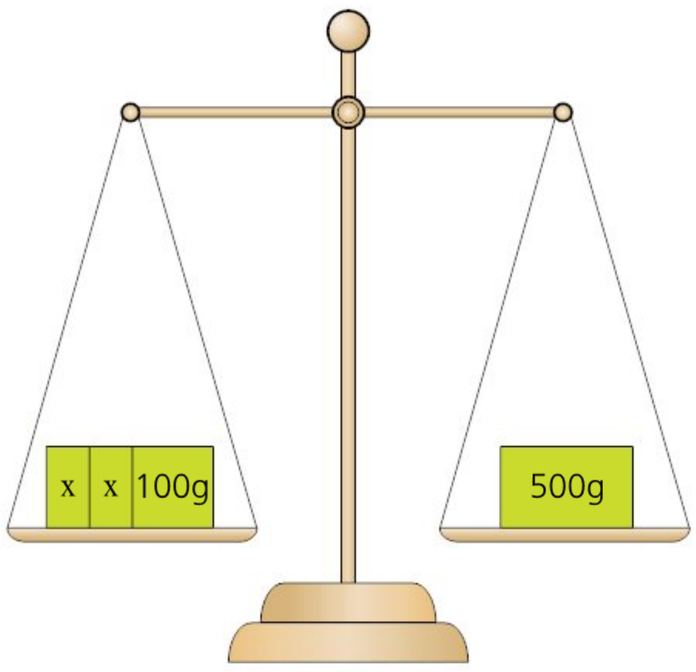




Other Vietnamese traders use scales to weigh their goods before they sell them to the public. A scale is a measuring instrument which tells you the mass of an object. A balance scale is a scale that uses a known mass (or masses) to find the unknown one.



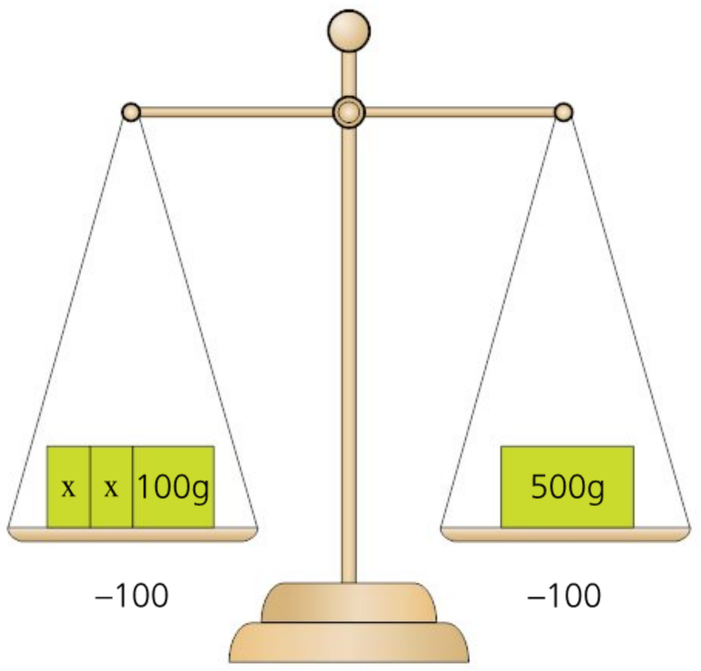
Suppose we have some objects of unknown mass x g that have been balanced on a pair of scales. We can use the visual tool of the balance to find the unknown mass.



We can form an equation

$2x + 100 = 500$

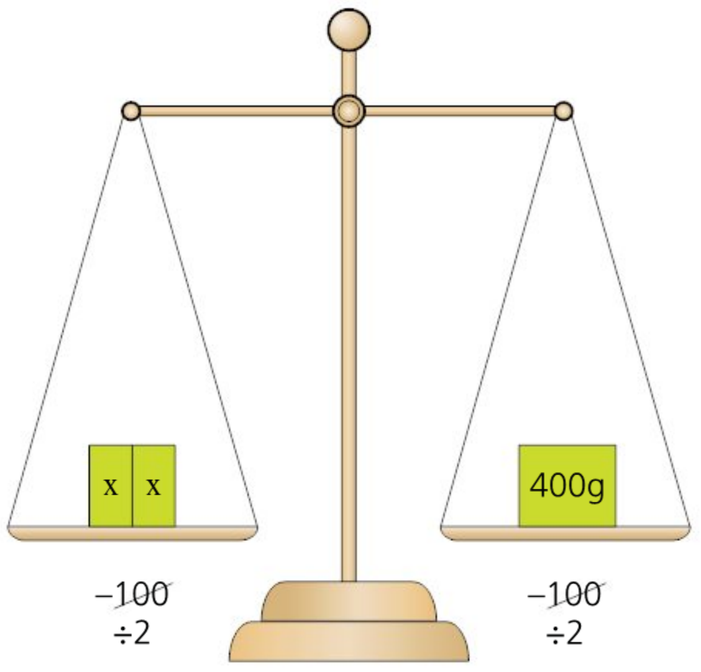
To isolate, or figure out the value for each x, we subtract 100 from each scale.



This gives the simplified equation

$2x = 400$

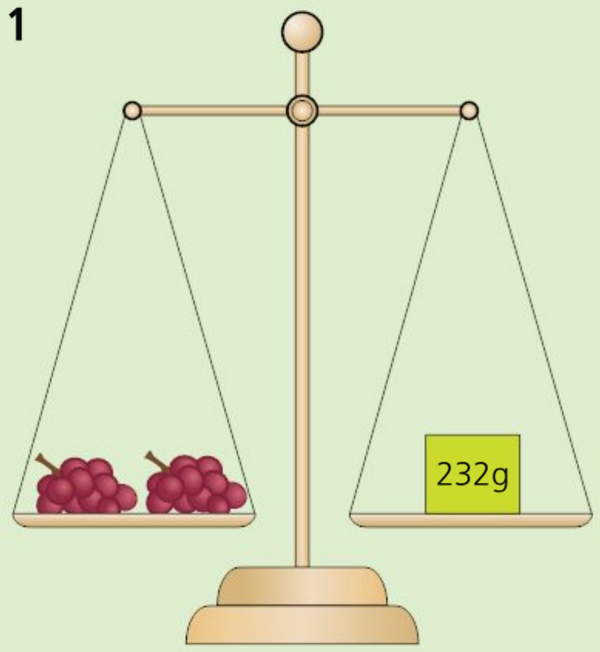
We can easily find that each x block has a mass of 200 g.



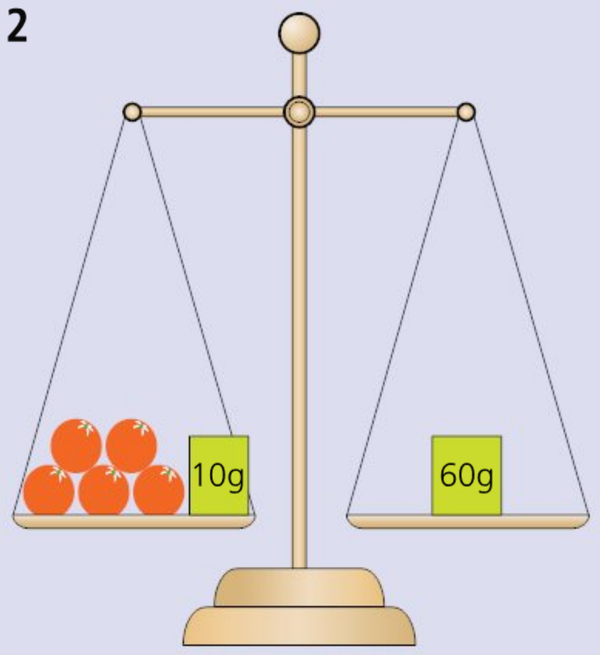
Try the following questions on your own. Make sure you **state** the equation you have formed and **show** your working. You can redraw the scales after each operation or not, whichever you find most helpful.

PRACTICE EXERCISE

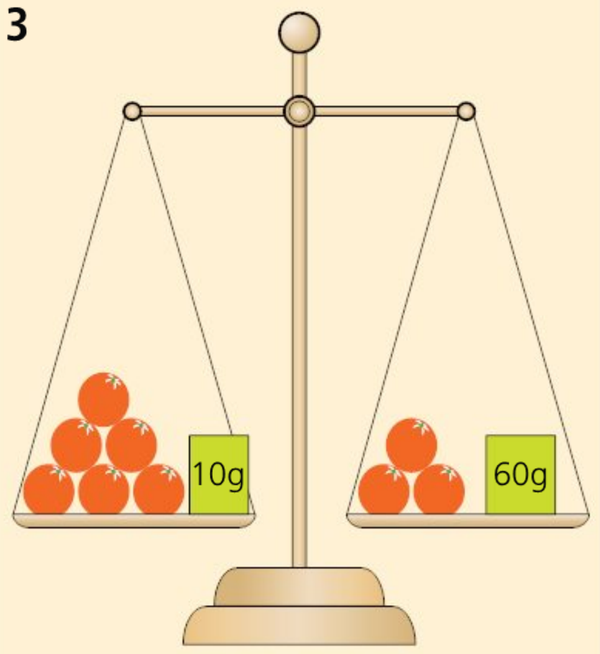
1



2



3



4 Make up some similar examples of equations in a visual form. Be creative!

SUMMATIVE ASSESSMENT: Demystifying mind-reading

ATL

- Critical thinking skills: Analysing and evaluating issues and ideas; Practise observing carefully in order to recognize problems

Think of a number …
now double it …



Occasionally online, you may come across websites or programmes that claim to able to read your mind.

These tricks often ask you to perform a series of algebraic operations and then they claim to be able to predict your answer, which is meant to amaze you!

Your task will be to figure out how these 'mind readers' work.

First watch this video: 'I know what your number is'
www.youtube.com/watch?v=cYGdA0SIMQo

There are lots of similar types of videos out there. Do they work? Are they right? How do 'mind readers' use numbers to do this?

Next let's look at the following website which makes a similar claim:
www.curiouser.co.uk/illusions/mindreader3.htm

The problem is outlined as follows:

Mind Reading Illusion

- Think of a number with 2 digits (e.g. 54).
- Subtract from this number its 2 digits (e.g. 54 – 5 – 4 = 45).
- Find the symbol from the chart below that corresponds to this number.
- **Concentrate** on the symbol for a few seconds and try to visualize the symbol in your mind.
- Click on the magic square…



Once you have performed the operations, the symbols matching each possible number are given below the 'magic box'.

99	I	98	M	97	U	96	U	95	{	94	O	93	I	92	I	91	R	90	N
89	b	88	R	87	d	86	i	85	S	84	S	83	R	82	m	81	x	80	{
79	J	78	U	77	J	76	M	75	d	74	J	73	b	72	x	71	N	70	z
69	x	68	h	67	I	66	i	65	d	64	6	63	x	62	v	61	O	60	{
59	J	58	m	57	I	56	I	55	I	54	x	53	T	52	^	51	J	50	{
49	h	48	T	47	m	46	f	45	x	44	M	43	b	42	z	41	o	40	6
39	6	38	{	37	R	36	x	35	h	34	z	33	U	32	6	31	d	30	^
29	S	28	m	27	x	26	O	25	R	24	I	23	J	22	I	21	o	20	i
19	I	18	x	17	R	16	U	15	J	14	o	13	R	12	b	11	i	10	o
9	x	8	d	7	T	6	v	5	N	4	O	3	N	2	m	1	6	0	x

Try this illusion on the website above. Did this one work? Repeat the illusion a few times.

You must figure out how the illusion works and explain it in a mathematical way. Find a general rule that tells you how the website can predict so perfectly.

Warning: a few clever tricks are used to keep you from spotting the pattern too easily!

Once you think you have a general rule, predict what you think the trick designer will say and why. Test your rule to see if you are right.

EXTENSION

Find some similar tricks online and see if you can explain them. Now you are an official magic debunker, what is your personal view of these 'illusions'?

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

Real-world interpretation

'Think-of-a-number' games have long been used to try and impress people. The person being tricked is performing mental mathematics, leading to a certain expected answer because of the operations used. What is your own personal view of this application of mathematics?

Some illusionists claim that these phenomena are spooky or magical. What is your personal view on this? Could mind-reading or magic be viewed negatively by certain people or cultures? How do you feel when you are tricked?

Is the power of mathematics being misused in illusions or is this just mathematics being used in a creative way?

Is mathematics more exciting if you don't know how it works or is magic less impressive if you do?

Discuss these big ideas as a class and use your own personal history and cultural background to enrich the conversation. If there is a variety of different opinions, why not have a debate to learn more about other people's opinions and perspectives?

Reflection

Use this table to reflect on your own learning in this chapter.						
Questions we asked	Answers we found	Any further questions now?				
Factual: What does commutative mean? How can we change equations with multiplication and division to **solve** them? What happens when there is more than one operation to 'undo' to **solve** the equation?						
Conceptual: What is the logic of the algebraic movements? Why is it important to define variables properly? How do we substitute into expressions or equations? Why do we substitute values into expressions?						
Debatable: How true to real-life are these scenarios for solving and substituting? Are these methods better than or more complex than using common sense to **solve** a number problem? Can personal and cultural differences affect how we see algebra?						
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?				
		Novice	Learner	Practitioner	Expert	
Self-management skills						
Communication skills						
Information literacy skills						
Learner profile attribute(s)	How did you demonstrate your skills as a thinker and a balanced person in this chapter?					
Balanced						

5 How do we measure up?



Measurement is expressed in various **forms** to communicate the **space** around or within an object

CONSIDER THESE QUESTIONS:

Factual: How can we represent size on paper? Are all triangles created equal? What is the difference between area and perimeter?

Conceptual: Can size be represented on paper? What difference does a degree make? How can you describe a shape? How are circles different from polygons?

Debatable: How do we define 'big'? Is it important to have a common language to communicate quantity? Can everything be broken down into a few basic shapes?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

PRIOR KNOWLEDGE

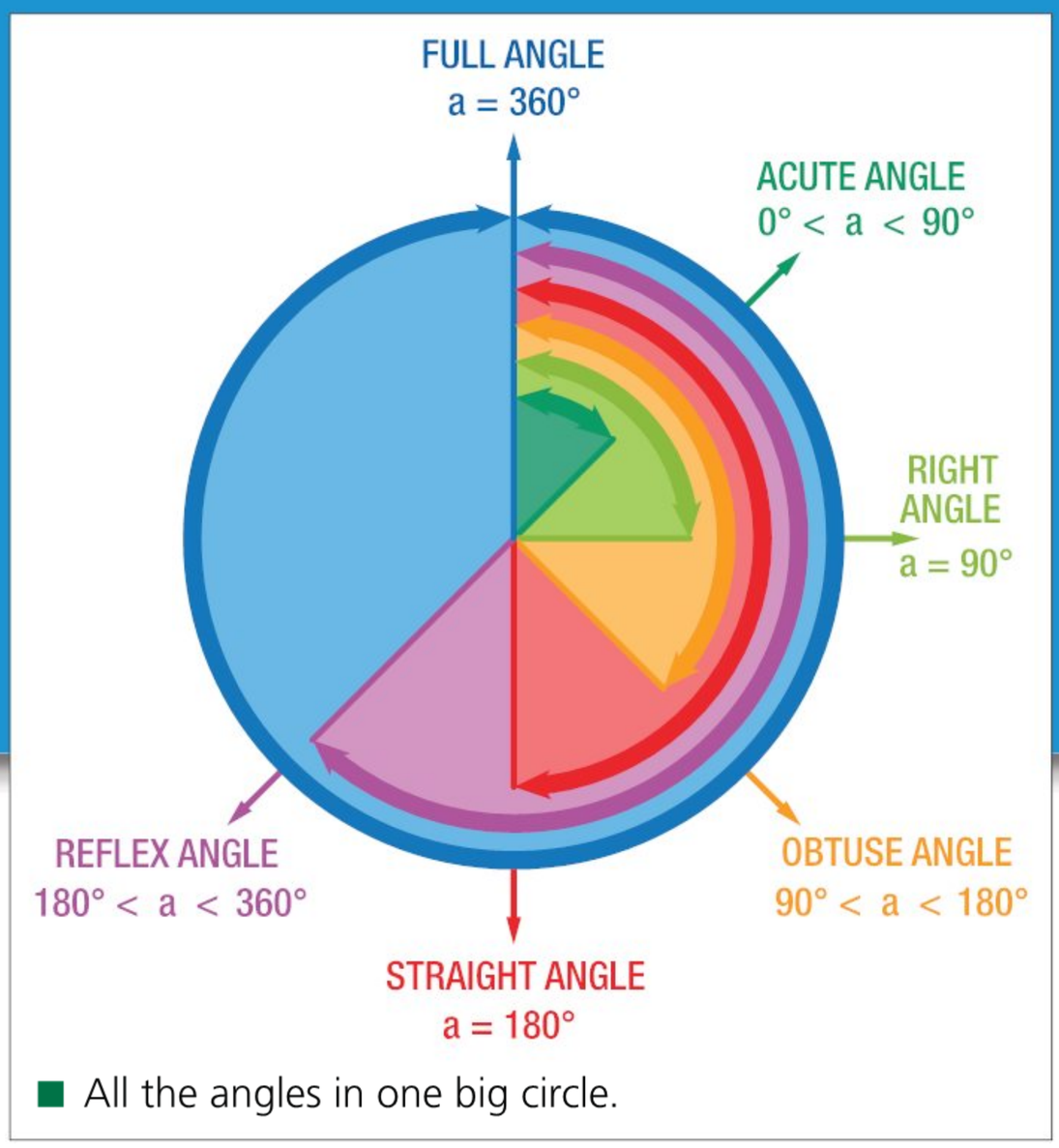
Reflect on what you already know about:

- using rulers, metre sticks, scales, etc. for measurement
- units of measurement you have heard of
- what is meant by area and perimeter and how are they calculated
- what an angle is and how to measure/draw/identify one.

IN THIS CHAPTER, WE WILL…

- **Find out:**
 - how to convert between units of measurement in the metric system
 - how to draw diagrams to scale and interpret scale diagrams
 - when and how to calculate area and perimeter of polygons, circles and composite figures, and how to measure and classify angles between two-line segments.
- **Explore:**
 - differences among various types of polygons and circles
 - the constant π (pi)
 - angles inside polygons.
- **Take action** by researching other units of measurement, and creating an angles flipbook.





- All the angles in one big circle.

These Approaches to Learning (ATL) skills will be useful …

- Communication skills
- Creative-thinking skills
- Critical-thinking skills
- Transfer skills
- Collaboration skills
- Organizational skills

We will reflect on this learner profile attribute …

- **Risk-taker** – We approach uncertainty with forethought and determination. We work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

Assessment opportunities in this chapter:

- **Criterion A:** Knowing and understanding
- **Criterion B**: Investigating patterns
- **Criterion C**: Communicating
- **Criterion D**: Applying mathematics in real-life contexts

KEY WORDS

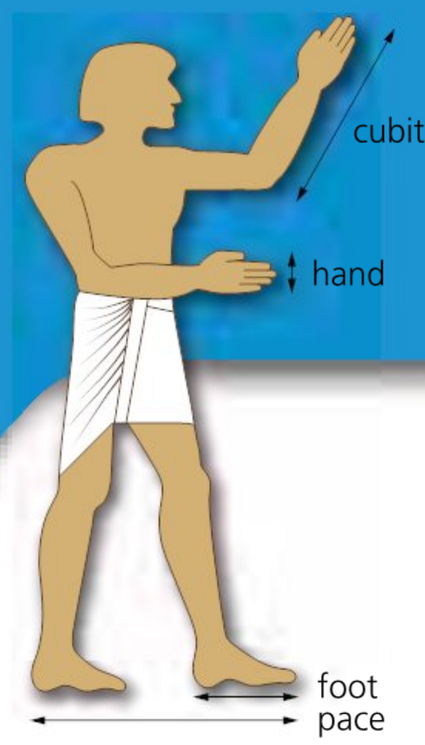
angle
length
mass
measurement
volume

THINK-PAIR-SHARE

Copy and complete the table by matching the units on the left with the measurement types on the right. Some measurement types may apply more than once.

Unit	Type of measurement
kilogram	Amount of liquid
megabyte	Mass
millilitre	Length/height/distance
ounce	Storage space
centimetre	
terabyte	
litre	
mile	
microgram	
kilometre	

How do we define ‘big’?



■ Ancient forms of measurement – why might these be inaccurate?

In 5000 BCE, long before money was invented, ancient peoples would exchange goods and services through bartering (trade). By this time, the people of the Indus Valley – today’s Pakistan, west India and north eastern Afghanistan – had already adopted a standardized (generally accepted) form of **mass** measurement by comparing various masses. For instance, one person might trade two sheep for one cow of equal mass. Babylonians used their arms and fingers to describe lengths, and empty containers were filled with plant seeds which were then counted to describe volume.

The need to agree on what was meant by ‘big’, ‘heavy’ and ‘long’ was crucial for bartering and construction. Egyptians and Mesopotamians eventually created the first unit of measurement, the ‘cubit’, using a cubit rod whose length extended from the elbow to the tip of the middle finger. This was not entirely consistent, as arm lengths vary, but gave a good overall estimate. Denominations of the cubit included palms, fingers and even finger nails. This led to the eventual creation of inches, feet and yards, and the Romans later added the mile (‘mille passus’, meaning 1000 paces).

DISCUSS

Can you find other units of measurement that were used in the past? Describe a few; select one and use it to recreate a real-life example.

By the 19th century, the most accurate units of measurement were what is now known as the ‘Imperial System’ from which ‘US Customary units’ are derived. They go something like this:

- 6 picas in an inch
- 12 inches in a foot
- 3 feet in a yard
- 1760 yards in a mile.

Today, in the United States, Myanmar and Liberia, these are still the official units of measurement. The rest of the world uses the metric system. This is a little easier to organize in our minds:

- 10 millimetres in a centimetre
- 10 centimetres in a decimetre
- 10 decimetres in a metre.

! Take action

Mathematical milestones

! Conduct a deeper inquiry into various other early mathematical milestones (numbers, simple probability, more units of measurement) and create a timeline to communicate your findings. Be sure to include the innovation or human need that led to the development of each new concept. What human needs do we have now that mathematics might address?

THE METRIC SYSTEM

Here is a **list** of the commonly used units of measurement in the metric system, depending on what it is you are measuring:

- metres – length
- grams – mass
- litres – volume.

Any subdivisions of these units use simple prefixes in front of that unit. For example, we have the metre to **measure** length or distance, and smaller units include centi**metres** and milli**metres**.

Let's focus on length first. Here, metres are the basic unit of measurement, and yet the ruler in your pencil case probably shows only centimetres and millimetres. Why do you suppose this is?

If metres were the only unit of measurement, then we would have to use extremely large and extremely small numbers on a daily basis. A drive to a relative's house may be 28000 m, and a waist measurement could be expressed as 0.74 m. These numbers are useable, but make things a little difficult to imagine – not to mention creating a mouthful to say! A 0.74 m waist *sounds* abnormally small! The basic unit of metres has, therefore, been broken down to make things more practical:

- millimetre $\frac{1}{1000}$ of a metre
- centimetre $\frac{1}{100}$ of a metre
- decimetre $\frac{1}{10}$ of a metre
- metre
- decametre 10 times longer than a metre
- hectametre 100 times longer than a metre
- kilometre 1000 times longer than a metre

Of these, the most commonly used units are the millimetre, centimetre, metre and kilometre. Take a moment to examine your ruler and note just how small a millimetre is. How many millimetres do you see in each centimetre? If your classroom has a metre stick, take a moment to examine that as well. Can you imagine 1000 of these metre rules laid end to end? How far would they go?

ACTIVITY: Let's practise!

ATL

- Creative-thinking skills: Apply existing knowledge to generate new ideas, products or processes

Measure each **line segment** below. State your measurements, first in centimetres, then in millimetres.



Can you think of a way to convert from centimetres to millimetres, and back again?

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding and Criterion B: Investigating patterns.

THINK-PAIR-SHARE

Indicate the sub-unit of measurement that would best be used to **measure** each of these variables:

- **your height**
- **the length of various insects**
- **the length of a wall in your house**
- **the thickness of the glass in a picture frame**
- **the distance to the nearest major city**
- **the thickness of a laptop.**

EXTENSION

What do bytes measure? How many bytes can be stored on:

- your phone
- your computer
- your email
- a memory stick
- an external hard drive?

What is a 'cloud' and why do people use them?

Is it important to have a common language?

Multiplying by 10 results in adding a zero to the end of a number, or moving the decimal point to the right (making the number larger). Dividing by 10 results in removing a zero from the end of a number or moving the decimal point to the left.

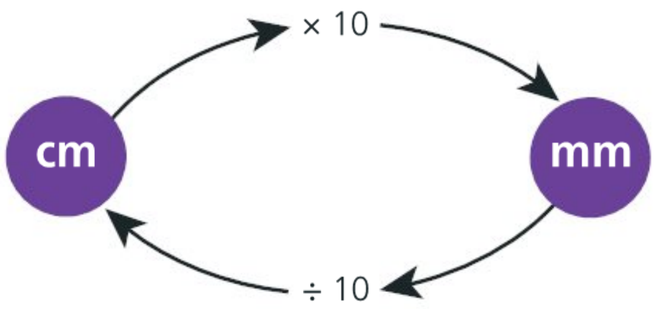
METRIC CONVERSION

You may have noticed in the previous exercise that centimetres are 10 times larger than millimetres (since there are 10 millimetres in each centimetre). This means that to convert centimetres to millimetres, we multiply by 10.

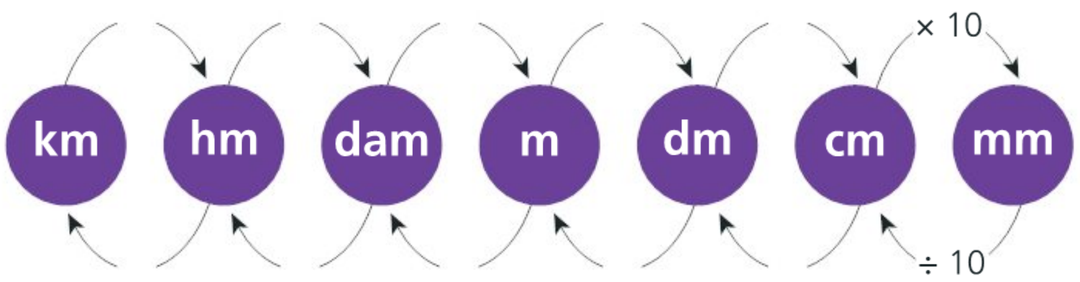
$$40\,\text{cm} = 40 \times 10\,\text{mm} = 400\,\text{mm}$$

Conversion flow chart

We've discovered how to change from millimetres to centimetres and back. In a conversion flow chart that operation would look like this:



Let's expand this flow chart to include **all** metric sub-units. We'll keep the portion from the flow chart above – can you complete the rest?



You can create a similar flow chart for conversion between sub-units of mass and volume.

Example

Convert 0.138 km to cm.

Solution

We are moving right five times on the flow chart, thus we multiply by 10 five times, and so we move the decimal point five places to the right:

$$0.1\overset{\frown}{3}\overset{\frown}{8}\overset{\frown}{0}\overset{\frown}{0}0\,0$$

Thus, 0.138 km = 13 800 cm

Notice that when we move from a large unit (in this case km) to a smaller one (cm), our number gets **bigger**. Why do you think this is? How can you do a quick check to see if your conversion makes sense?

EXTENSION

Neither kilometres nor centimetres are an appropriate unit for the number given in the example above as people generally prefer smaller whole numbers. What might be a more appropriate unit to use and why?

Are 4 and 4.00 equal?

If ever you see zeros after a decimal, don't be too quick to dismiss them! A measurement like 4.00 L, for instance, indicates that the measurement was precise to the nearest hundredth. You can be sure that the amount is not 'somewhere in the range of' 4 L – not 4.1 L, nor 4.44321 L for that matter.

Example

A drink recipe requires 1.5 L of lemonade, but the store has run out of cartons and only has 250 mL cans. How many cans will it take for the recipe?

Solution

Looking at our conversion flow chart from page 102, we must multiply by 10 three times – that is, move the decimal point three spaces to the **right** to convert litres to millilitres:

1.5 0 0 0 0 0

This comes to 1500 mL.

If the cans each contain 250 mL, then we need

$\frac{1500}{250}$ = 6 cans of lemonade

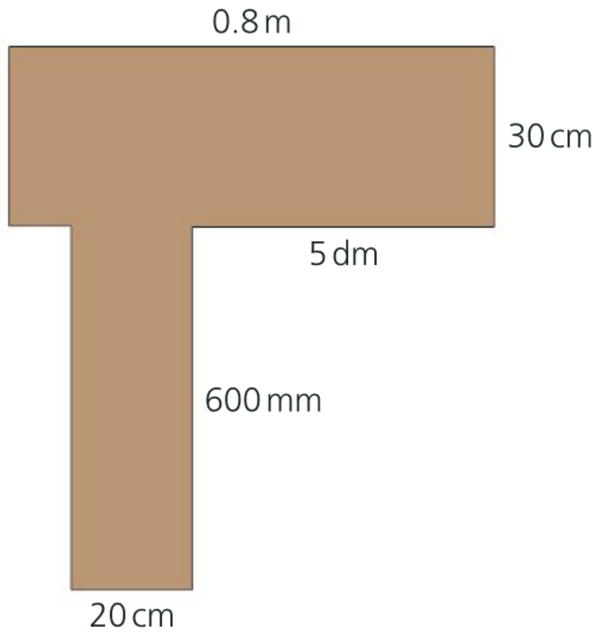
ACTIVITY: One common language

ATL

- Communication skills: Use appropriate forms of writing; Understand and use mathematical notation

You may **recall** that, when comparing fractions, we needed a common denominator before we could do the **comparison**. Similarly, when comparing units of measurement, we should always use the same sub-units.

1. **Circle the larger quantity in each pair:**
 - **7 m** **700 mm**
 - **3 km** **280 000 cm**
 - **30 L** **30 000 mL**
 - **10 dg** **0.1 hg**
 - **400 MB** **0.4 TB**
2. **Copy and relabel the diagram below so that all measurements are of the same unit. Why did you choose the unit that you did?**



Assessment opportunities

In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

ACTIVITY: Walking the wall

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations; Combine knowledge, understanding and skills to create products or solutions

The Great Wall of China runs for 21 196 km.

1. **Calculate the mean (average) metres of length that were built per year if construction began in 500 BCE and ended in 220 BCE. What would be a good first step before determining this value?**
2. **If an average person's pace (footstep) is 35 cm, how many paces are required to walk along 10% of the wall? Can you estimate how long that would take? How long would it take to walk along the entire wall? What are you using as a basis for time in estimating this?**
3. **Until its first formal measurement in 2012, the length of the Great Wall of China was estimated to be 8000 km. What percentage of the actual length is this?**

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

How can we represent size on paper?

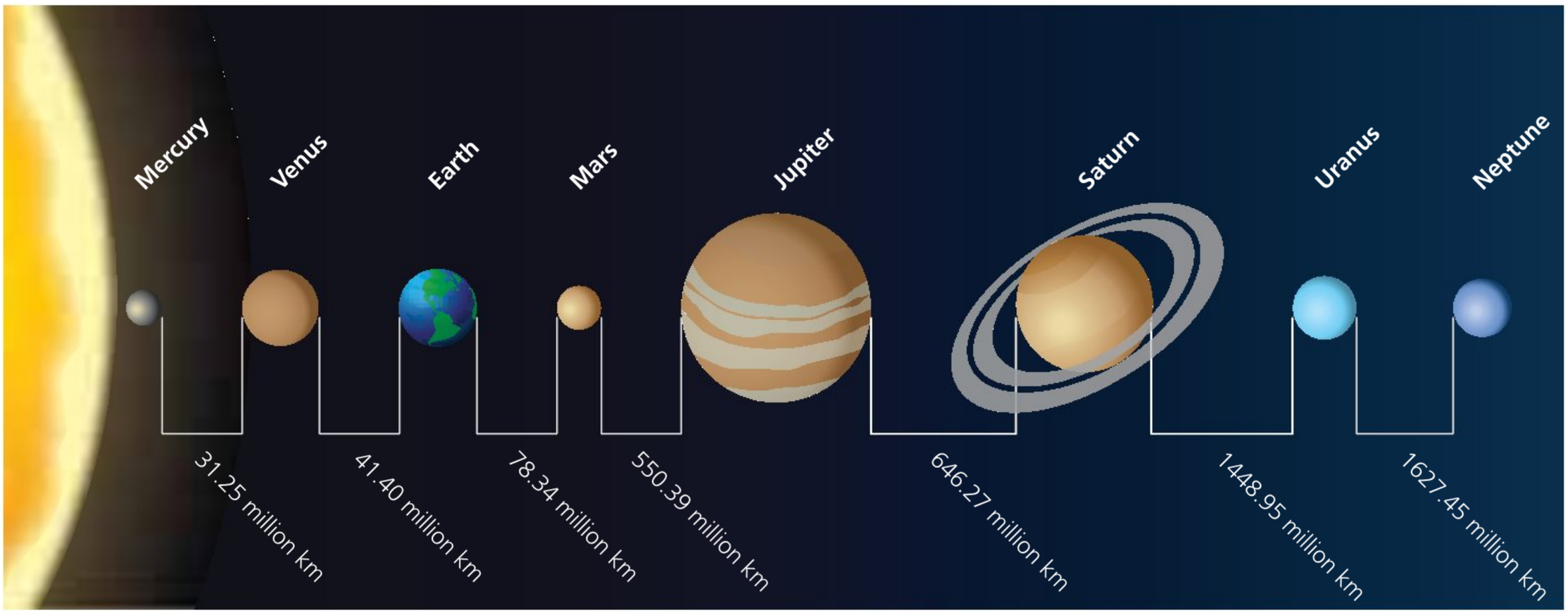
WHAT MAKES YOU SAY THAT?

What is wrong with the Solar System diagram shown below? What do you see that makes you say that?

EXTENSION

1 AU is the distance from the Earth to the Sun. How many AU from Earth to Mars?

SCALE DRAWINGS



■ Planets and their distances from one another

Scale diagrams and models are used to show objects or spaces as accurately as possible in a smaller (or larger) way. This technique is widely used in the world of architecture and urban planning, but also in technology (when zooming in and out of images, for example), and in various other contexts. Why do you think an architect might want to design an accurate scale drawing before beginning construction? What do you need to explain to the man in the video clip below?

www.youtube.com/watch?v=mBNom46c4tQ



ACTIVITY: The shortest distance

ATL

- Communication skills: Read critically and for comprehension; Make inferences and draw conclusions; Use and interpret discipline-specific terms and symbols; Understand and use mathematical notation

The map shows Alexander the Great's conquests from 334–328 BCE. He took a very indirect route to arrive at Gandhara. What is the shortest distance from Macedonia to Gandhara? Why do you suppose Alexander the Great veered from this path?



Alexander the Great's conquests

Using a ruler

Measure the distance from the word Macedonia, to the word Gandhara. If every centimetre represents 300 km, as described in the **scale** at the bottom left, then how far is the distance you measured?

Hint

1 cm = 300 km, so your measurement multiplied by 300 will be the distance you measured in kilometres!

Using a compass

An alternative way to **determine** the distance is to line a compass up with the 0 and 300 in the scale and count the number of times this interval fits from Macedonia to Gandhara.

Which method do you believe to be more accurate? Why?

EXTENSION

Can you think of other methods that might be more accurate? How could you check these? Why would they be more accurate? Would Alexander the Great have been able to **apply** your method in ancient times?

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

Hint

If you are confused by what is represented in the ratio – remember that the bigger number represents the bigger image, whether that is the actual image or the scale drawing.

- If you are reading a diagram that represents something **larger**, the numbers need to get larger, and we do this by **multiplying**.
- If you are reading a diagram that represents something **smaller**, you need to make the numbers smaller by **dividing**.

INTERPRETING SCALE DIAGRAMS

Scales are labelled less frequently in the convenient manner we saw in the Alexander the Great map, and more commonly with a **scale factor**. The scale may look something like this: 5 : 1

Example

The image below shows the magnification (enlargement) of what is believed to be the first ever monetary currency, a coin from the Lydian empire, developed in 600 BCE from a combination of silver and gold. The image is to scale, with a ratio of 5 : 1. This means that every 5 mm on the page represents 1 mm in real life. Calculate the width (shown by the line above the image) of the actual coin.



■ Lydian Coin from 600 BCE – the first ever currency

Solution

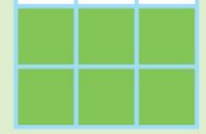
The coin in the image measures 65 mm across. Because the image is larger than the actual coin, we will need to **divide** to get the dimensions of the real coin:

65 mm ÷ 5 = 13 mm

CREATING SCALE DIAGRAMS

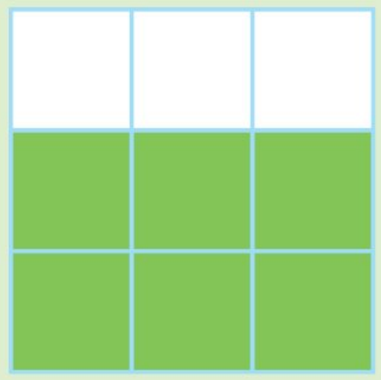
Example

Greta, Jan and Emil are asked to draw the rectangle that is represented by the shaded area in this figure.

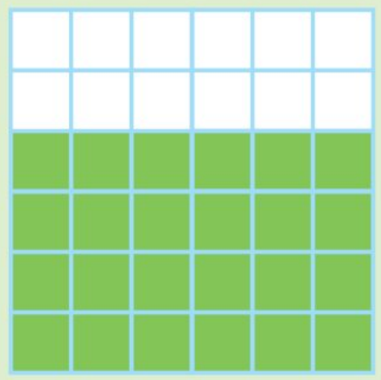


Solution

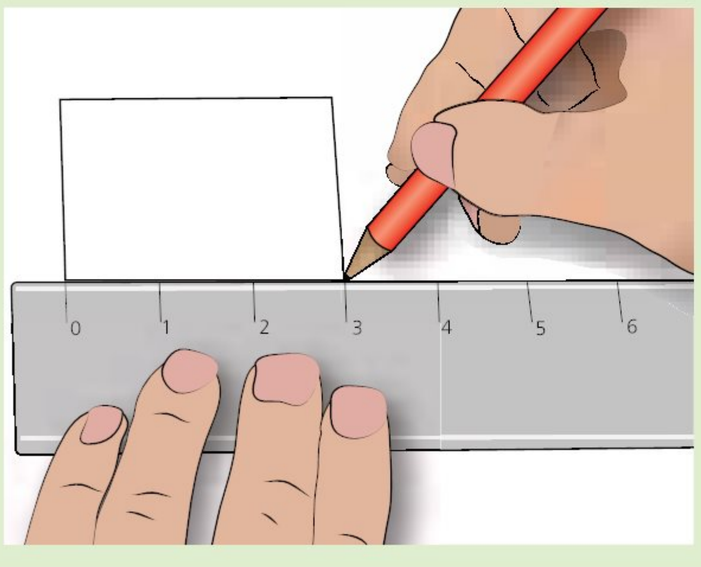
Greta notes that, since the ratio is 1 : 2, every 0.5 cm square on the grid can represent a 1 cm square on the image grid.



Jan notes that the grid is 3 × 3, so he draws a 6 × 6 grid of small squares that each measure 0.5 cm by 0.5 cm.



Emil simply measures the rectangle and notes that the length is 1.5 cm and the width is 1 cm. He multiplies these two numbers by 2. He obtains a larger rectangle by multiplying the lengths of the sides by the scale factor.



Links to: Individuals and societies: History

Archaeologists excavate historical sites in search of artefacts and other remains that will help them better understand history and verify proposed facts. Why do you suppose they would use grids when doing this, such as in the image below?



An archaeologist's grid system

ACTIVITY: Scale drawing of real-life object

ATL

Communication skills: Organize and depict information logically

Select any object in the room that is bigger than your paper. **Measure** every dimension and record it on a rough sketch. **Draw** an image of the object that is to scale, and **state** your scale factor. Do you need gridlines for this? Why or why not?

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion C: Communicating.

MEET A MATHEMATICIAN: NIKOLA TESLA (1856–1943)

Learner profile: Risk taker

Long before texts and GPS satellites entered our lives, a Serbian–American mathematician and physicist wrote an article on wireless interplanetary communication. Although he risked his well-respected reputation in doing this, Nikola Tesla's vision does not seem so far-fetched today.

> '… there is a solid foundation for a systematic attempt to establish communication with one of our heavenly neighbours, as Mars, which through some inventions of mine is reduced to a comparatively simple problem of electrical engineering. Others may scoff at this suggestion or treat it as a practical joke, but I have been in deep earnest about it ever since I made the first observations at my wireless plant in Colorado Springs from 1889 to 1900.'



■ Nikola Tesla

In 1901 Tesla began work on a global wireless communication system, funded by J.P. Morgan. With this project, Tesla intended to provide free electricity and information exchange to the world. When J.P. Morgan realized the electricity would be freely available and turn very little profit, he withdrew his funds. Telsa carried on – incurring great debts to finance the project. He risked everything to build the transmission tower at the famous Wardencliffe site, but was finally forced to declare bankruptcy and never saw it come to fruition. As we now know, wireless technology has caught up with Tesla's vision.



■ A Tesla car

While Tesla did not establish interplanetary communication in the late 19th century, he was able to transmit energy without wires for miles, at a time when many households still lit their rooms with candles. He also changed the world's thinking about the possibilities of wireless technologies – including the very near possibility of interplanetary communication which has since been achieved. Nikola Tesla also created the 'Tesla coil', which was essential for sending and receiving radio waves and led to the first wireless communication from England to Canada.

A risk-taker to his final days, ending his life in poverty due to his constant debts to finance his world-changing innovations, Tesla has left a strong legacy. The currently popular Tesla Roadster uses an a.c. motor descended directly from Tesla's original design. The standard unit for magnetic flux density is the tesla, where 1 T represents one kilogram per second squared per ampere.

Tesla's discoveries also include:

- fluorescent light
- the laser beam
- wireless communications
- wireless transmission of electrical energy
- remote control
- robotics
- Tesla turbines
- vertical take-off aircraft
- alternating current induction motor – one of the greatest discoveries of all time!

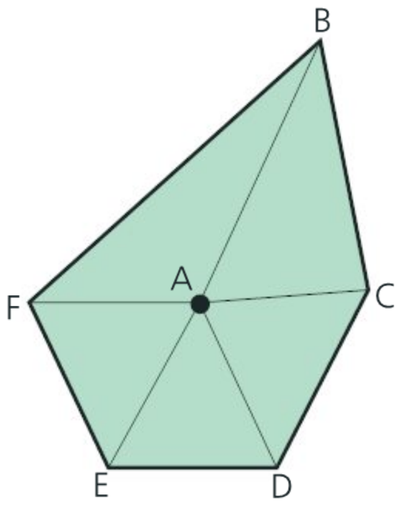
He also held 700 other patents throughout the world for his inventions.

▼ Links to: Sciences

Try this search term: **'10 best ideas for interplanetary communication'**. Why are so many scientists also mathematicians?

What difference does a degree make?

NAMING ANGLES

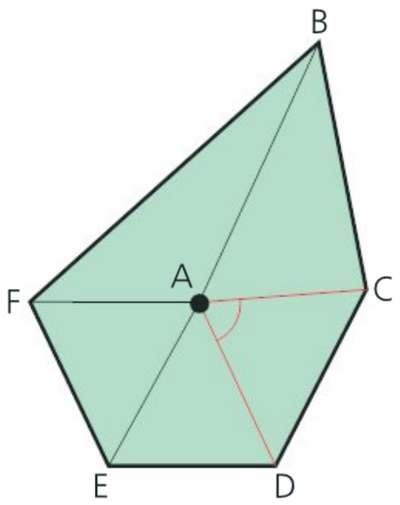


Consider this question:

State angle A shown in the diagram here.

What makes this a difficult question?

It is crucial to have a common system of angle reference to avoid miscommunication. To name an angle, we must **state** the starting point, the vertex (where the actual angle measurement takes place) and the ending point. In the diagram above ∢A could have been any one of five angles, but if we specify angle DAC, ∢DAC, we can all zone in on the exact same angle.

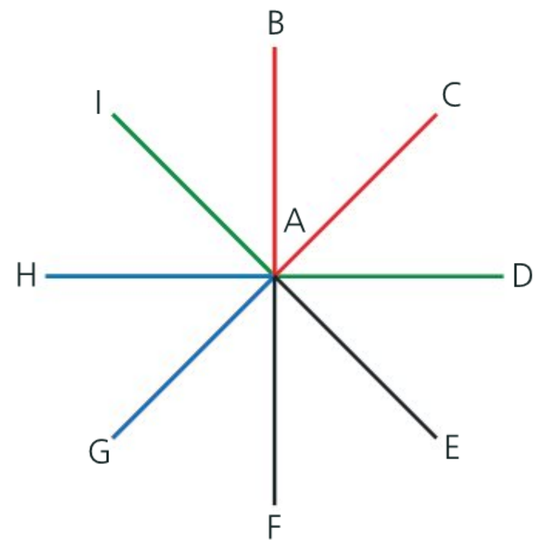


ACTIVITY: Name that angle or line segment

ATL

- Communication skills: Read critically; Use and interpret a range of discipline-specific terms and symbols; Understand and use mathematical notation

State the name of the angle between each pair of line segments with the same colour.

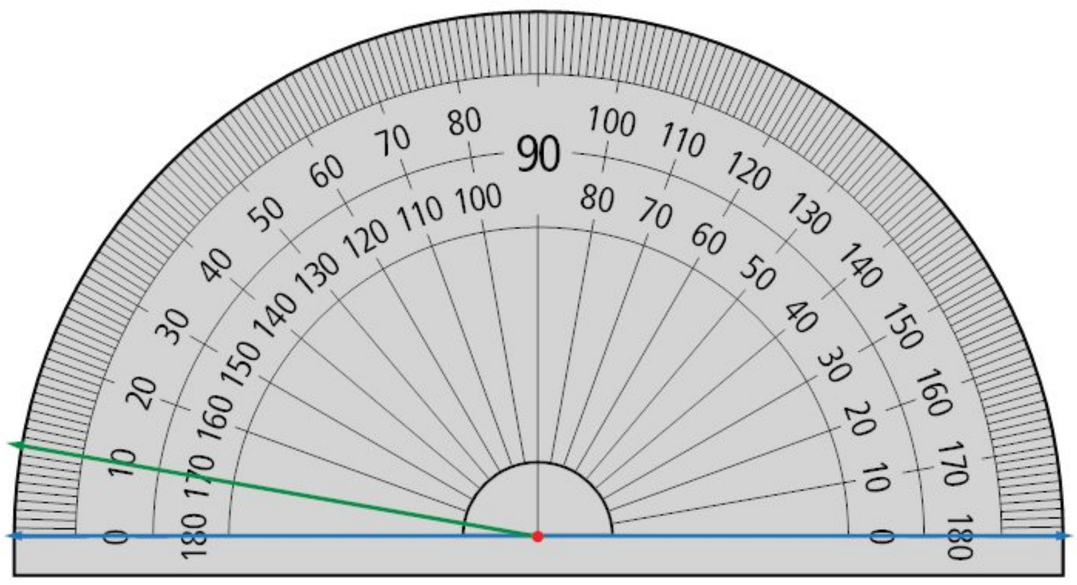


Assessment opportunities

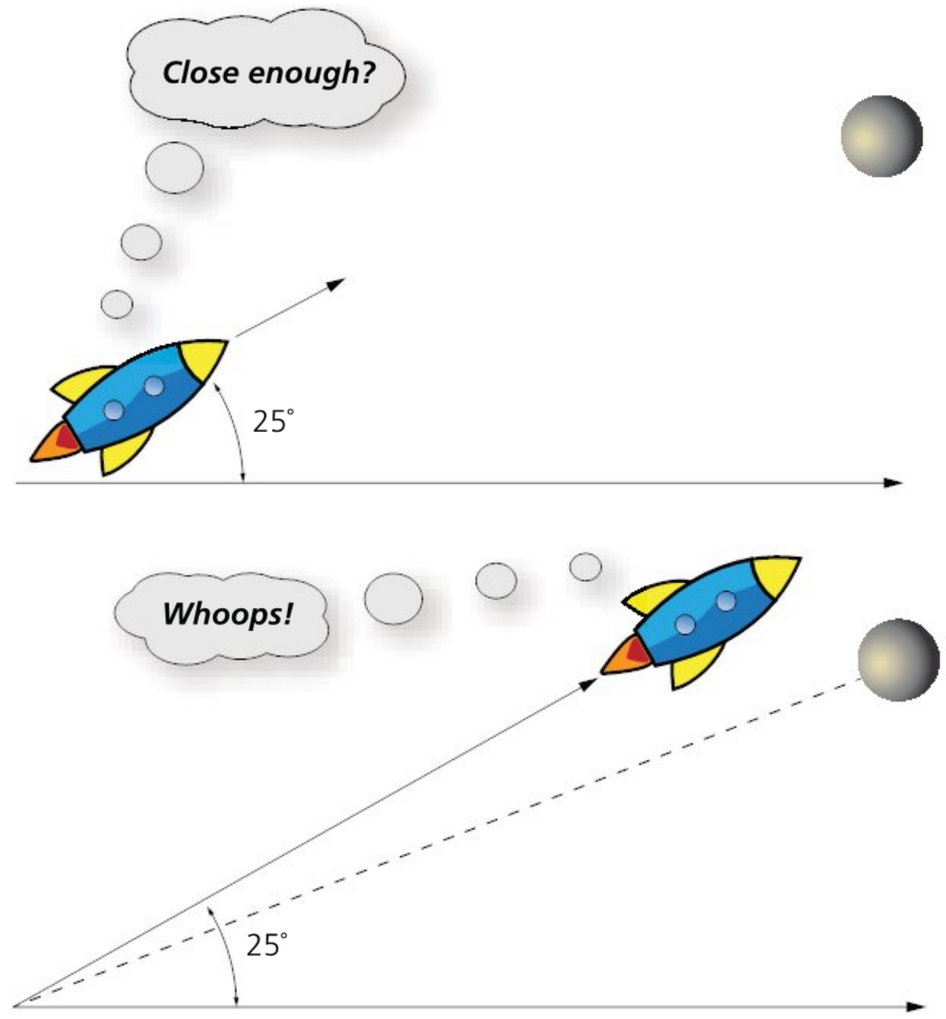
In this activity you have practised skills that are assessed using Criterion C: Communicating.

USING YOUR PROTRACTOR

Take a close look at the **protractor** below. What happens to the distance between the 10° line and the horizontal (in blue) as you move from the centre (red point) outwards?

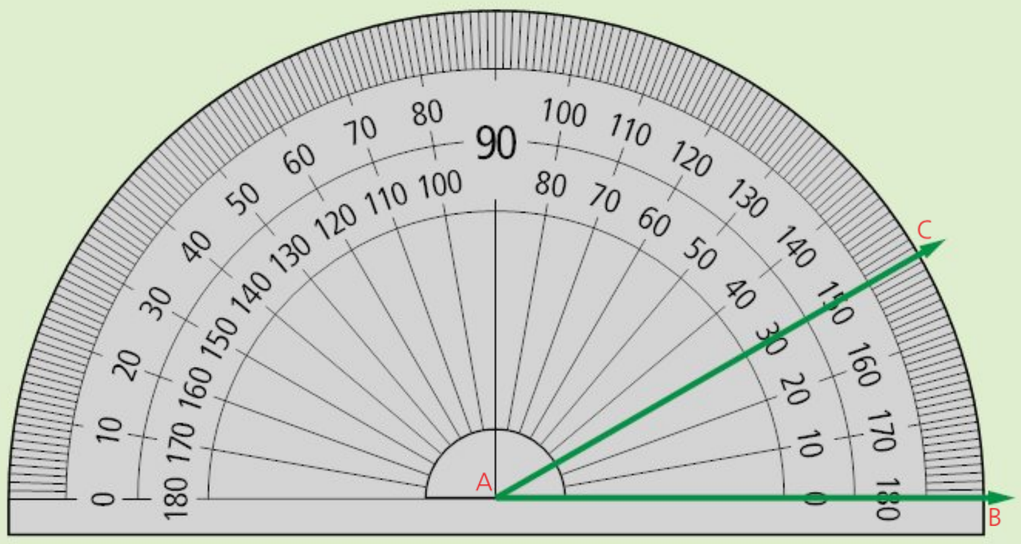


Imagine being in a spaceship destined for Mars and your launch is set at 25° (25 **degrees**) from the ground, while Mars is at 24°. How might this deviation of 1° affect your mission?



Although looking at your protractor the difference between one degree and the next may seem insignificant, if you extend the lines that form these angles, the difference gets bigger and bigger! Let us take a moment to familiarize ourselves with protractors.

Measure angle BAC.



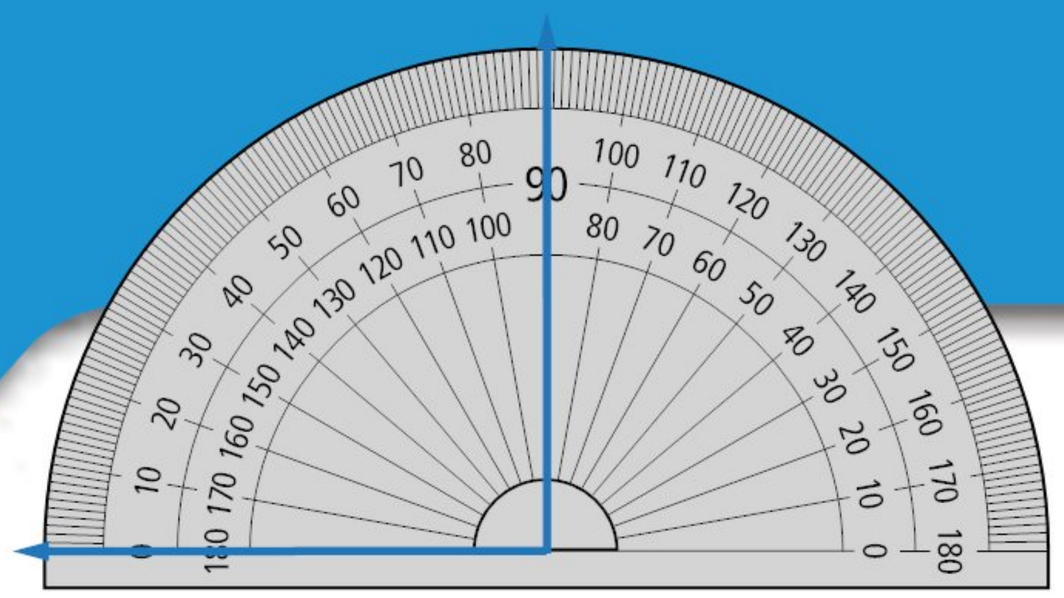
1 Line up the origin of the protractor with the point – or 'vertex' – of the angle.

2 Rotate the protractor until its baseline matches up with one of the angle's arms.

3 Follow the opposite angle arm to the measurements on the protractor's arc. But wait! There are **two** measurements!

- If the angle is smaller than 90° (**acute**), use the smaller measurement.
- If the angle is larger than 90° (**obtuse**), use the larger measurement.

Looking at this angle, it is clearly smaller than a 90° angle, so we will use the 30° instead of the 150°.



TYPES OF ANGLES

One of the most important angles of all is the **right angle**. It is so important, it has its very own symbol! Why do you think the symbol ⌞ was chosen to represent a right angle?

A right angle measures exactly 90° and is the angle formed by a vertical line at the origin of your protractor. It is essential in many facets of life, especially in construction.



■ Right angle – Taj Mahal, India.



■ Oops! Not a right angle – Pisa, Italy.



■ An obtuse angle at the peak of the pyramid – larger than 90°, but smaller than 180°.



■ 180° can be thought of as a straight line or a switch to the opposite direction.



■ Reflex angle – between 180° and 360°.

ACTIVITY: What type of angle is it?

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations
- Communication skills: Read critically

Classify each angle in the images and mark any examples you see of right angle, obtuse, acute, reflex or full revolution angles. State what would happen if an angle was changed to another of a different classification (for example, what if the acute angle formed by the chopsticks was obtuse?).



Assessment opportunities

In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding and Criterion D: Applying mathematics in real-life contexts.



Full circle or full revolution – 360° begins and ends in the same place.

Take action

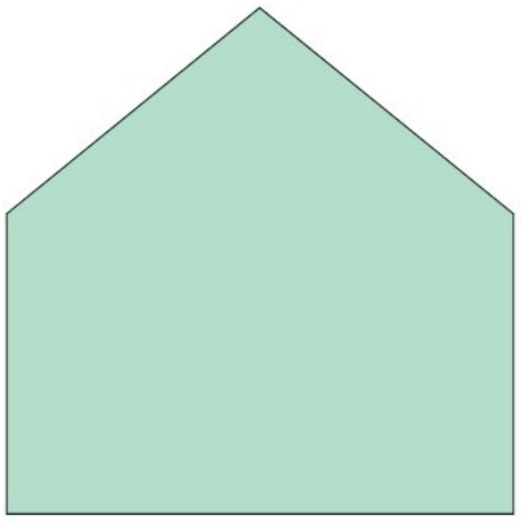
! In 2007, Disney Studios added a flipbook to the introductory sequence of its movies prior to the appearance of the Disney logo, as a nod to early animation: **www.youtube.com/watch?v=KW_K9UJ6eKs**. Flipbooks are made by drawing a series of pictures that differ only slightly from page to page, so that when you flip through them, an image seems to move. Your task is to **design** a flipbook of two line segments that begin on top of one another and, as you flip through the book, one line segment moves away from the other at regular intervals from 0 degrees to 360 degrees. Highlight any turning points in your animation – points at which your angle changes from acute to right angle, to obtuse, and so on.



Disney flipbook

How can you describe a shape?

All **closed** two-dimensional shapes without any curves in them are called **polygons**. A closed shape is one that does not contain an opening.

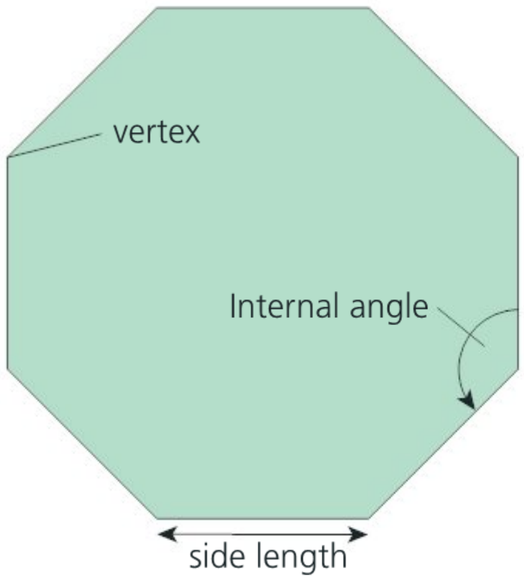


■ Closed shape



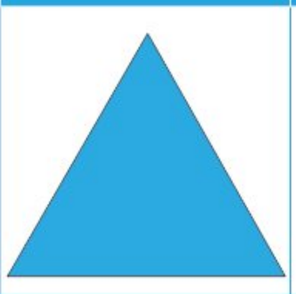
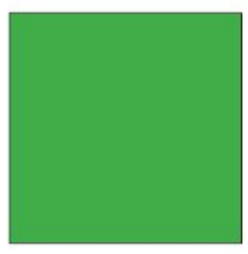
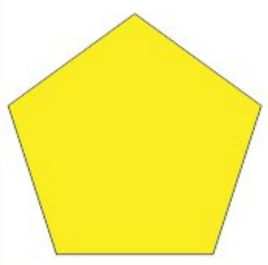
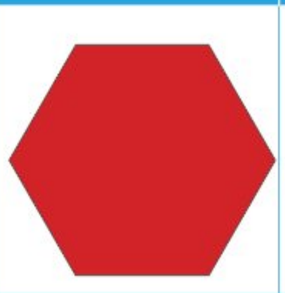
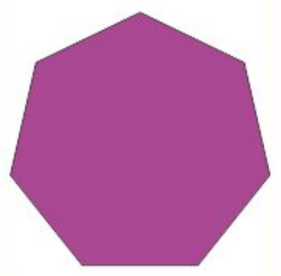
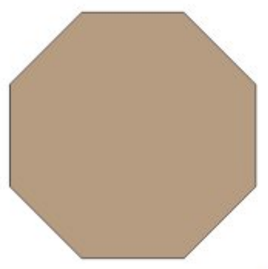
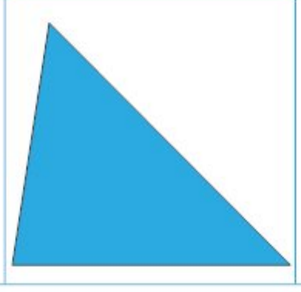
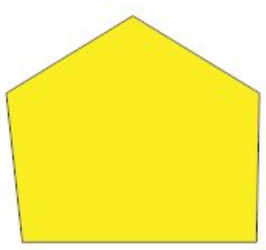
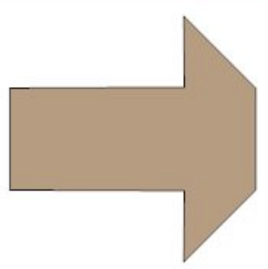
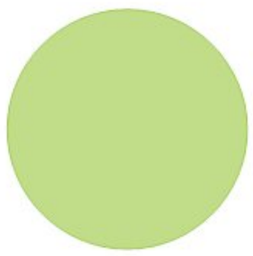
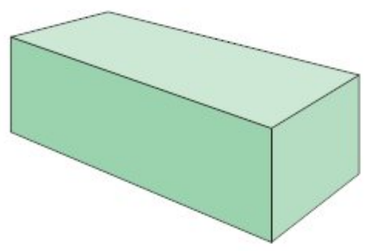
■ Open shape

ANATOMY OF A POLYGON



■ Regular octagon and its properties

If a polygon happens to have all equal side lengths with all equal **internal (interior) angles**, it is referred to as a **regular** polygon. Otherwise it is the opposite – an **irregular** polygon.

A polygon can have three or more sides	3 sides Triangle	4 sides Quadrilateral	5 sides Pentagon	6 sides Hexagon	7 sides Heptagon	8 sides Octagon
Regular polygons All sides are equal length and all internal angles are equal						
Irregular polygons Any polygon that is not regular						
Not polygons	Circles 	Any shape that includes a curve 	Any shape that isn't closed 	Three-dimensional objects 		

DISCUSS

Is a circle a polygon?

ACTIVITY: Forensic sketch artist

ATL

- Communication skills: Give and receive meaningful feedback; Use intercultural understanding to interpret communication; Make inferences and draw conclusions; Use and interpret a range of discipline-specific terms and symbols

A forensic sketch artist interviews witnesses and victims of crime, gathering information about the appearance of a suspect, and draws a sketch to match that description.

You are the witness to a crime, and the suspect is a polygon which you can't **identify** by name!

Your task is to **describe** this polygon to a sketch artist (a partner) and hope that, as a team, you will produce a replica that is good enough to help find the suspect.

Select a polygon – it can be one from the table on page 112, or an irregular polygon of your own devising. **Describe** it to a friend using any of the following terms that **apply**:

- **vertex**
- **unequal**
- **acute**
- **shorter**
- **equal**
- **obtuse**
- **longer**

You may not name your shape or **state** whether it is regular or irregular.

Your friend will draw the shape based on your description, much like a police sketch artist does as witnesses **describe** suspects. Irregular polygons will prove especially challenging! When finished, you and your partner can switch roles.

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion C: Communicating.

EXTENSION

Copy and complete the table below which lists the prefixes for polygons with five or more sides:

Number of sides	Prefix
Five	Penta-
Six	Hexa-
Seven	
Eight	
Nine	
Ten	
Eleven	
Twelve	

ACTIVITY: How do internal angles add up?

ATL

- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions
- Creative-thinking skills: Use brainstorming and visual diagrams to generate new ideas and inquiries, Apply existing knowledge to generate new ideas, products or processes

1. **Draw three triangles of varying side lengths and angles.**
2. **Measure and label the angles within each triangle.**
3. **Find the sum of the angles within each triangle by adding all three together.**
4. **Repeat this process with a square, and with other polygons. What happens each time you increase the shape by another side?**
5. **Describe any pattern you see.**
6. **Suggest a general rule to determine the sum of all interior angles in any regular polygon.**

Assessment opportunities

This activity can be assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

Are all triangles created equal?

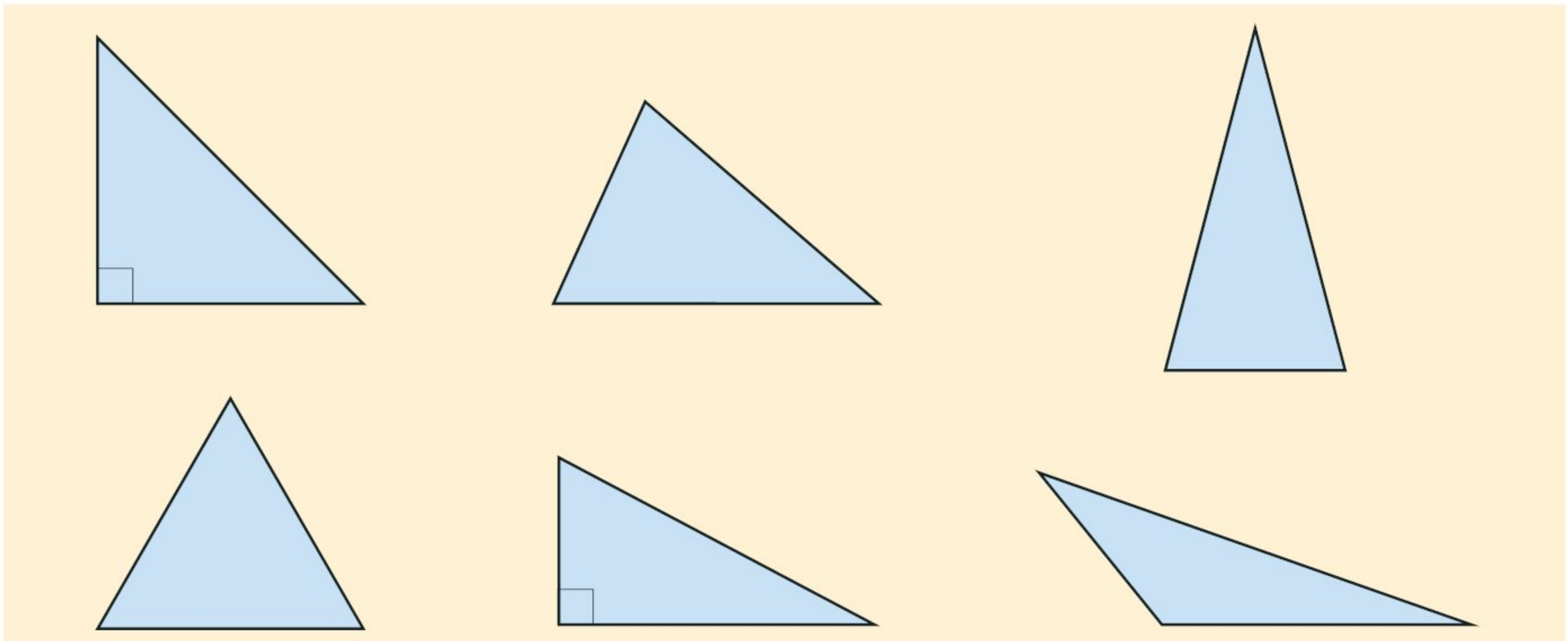
Of all the shapes studied in mathematics, triangles receive a special mention. They are so vital that there is an entire branch of mathematics devoted to them; it is called **trigonometry**. Trigonometry is crucial to GPS systems, computer graphics and game development, ocean and aircraft navigation, locating stars, planets and other celestial objects, CAT scans and ultrasounds, predicting weather patterns, land surveying, engineering and a wide variety of other contexts. Before we can begin to look at trigonometry, we need to understand some basics of triangles – starting with how we classify them.

WHY DO I NEED TO KNOW HOW TO NAME A TRIANGLE?

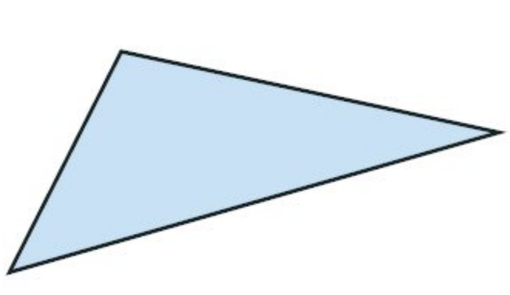
Once you can classify triangles into their respective categories, it becomes much easier to **calculate** missing side lengths and angles – as you will see in future mathematics courses. Let's get started!

PRACTICE EXERCISE

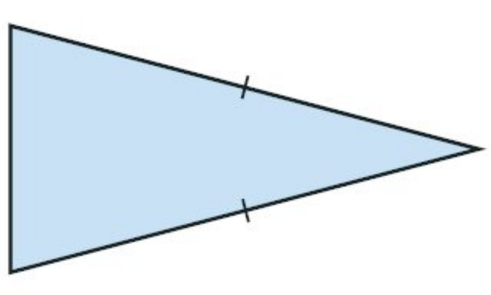
Measure the side lengths of the triangles below, and **state** the number of acute, obtuse and right angles. Which would you group together, and why? Is there more than one way to group them?



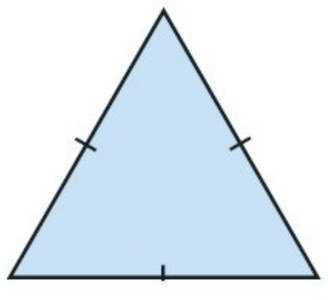
There are three possible ways to **label** a triangle based on the **length of its sides**:



Scalene: 0 equal sides

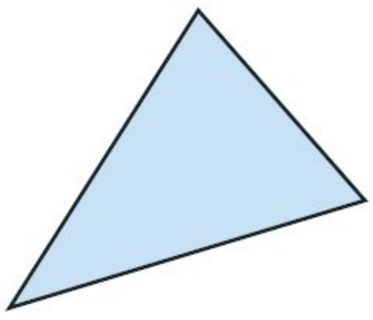


Isosceles: 2 equal sides

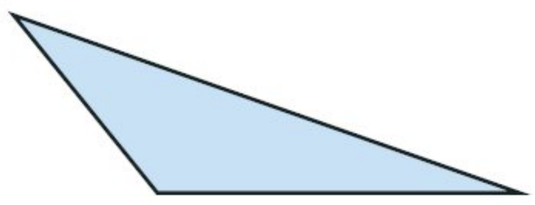


Equilateral: 3 equal sides

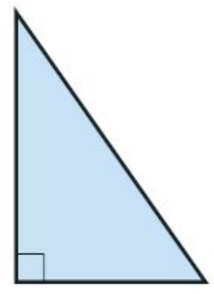
How was your classification scheme different from this one? What did you think was important or relevant? Are there other ways to sort these triangles into order? There are three possible ways to **label** a triangle based on the **size of its angles**:



All acute angles: acute triangle



1 obtuse angle: obtuse triangle



1 right angle: right-angle triangle

PRACTICE EXERCISE

1 **Draw** a small and a large acute triangle and measure the sides and angles to **verify**.

2 **Draw** a small and a large isosceles triangle and measure all angles and sides to **verify**.

3 **Draw** two right-angled triangles, one inside the other and measure all sides and angles to **verify**.

4 **Draw** an isosceles obtuse triangle inside an equilateral acute triangle and measure all sides and angles to **verify**.

ACTIVITY: Too obtuse?

ATL

- Creative thinking skills

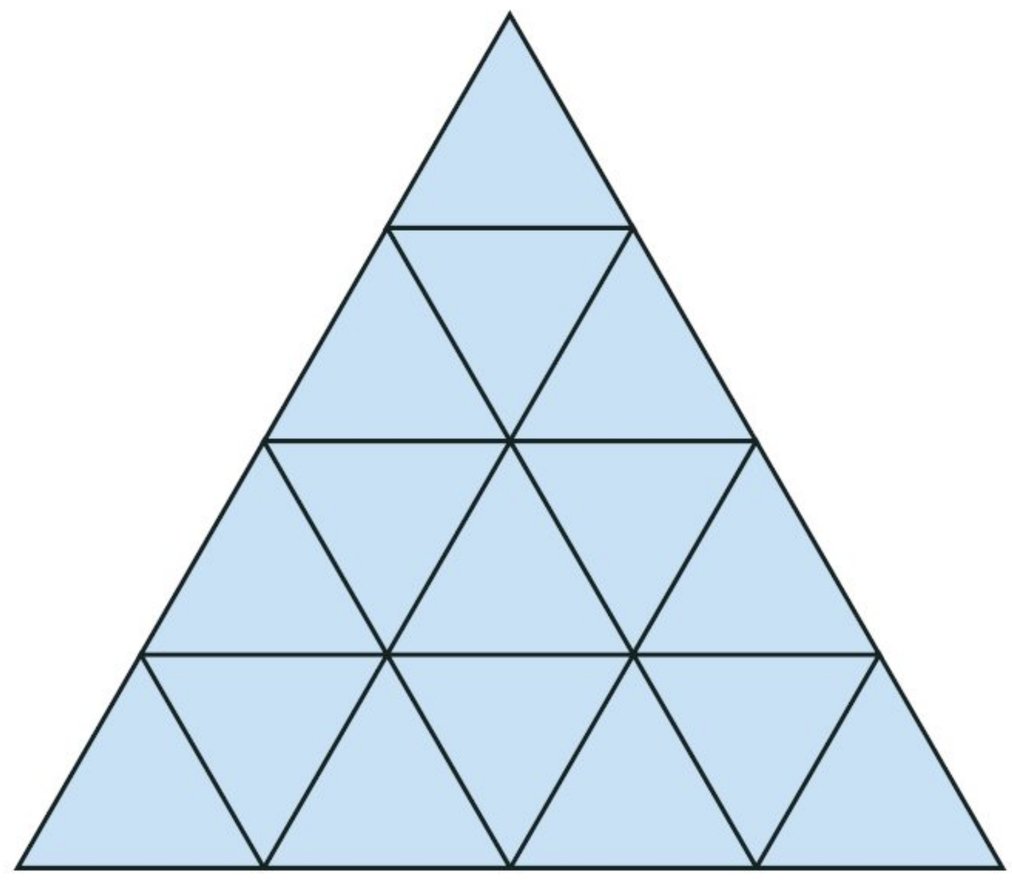
Assign two labels to each of the triangles in the Practice exercise on page 114 – one to **describe** the lengths of their sides and the other to **describe** the sizes of the angles within. Using the results from Activity: How do internal angles add up? on page 113, **discuss** whether it is possible to have a triangle that has two obtuse angles.

Assessment opportunities

In this activity you have practised skills that may be assessed using Criterion B: Investigating patterns and Criterion C: Communicating

SEE-THINK-WONDER

How many triangles can you see in this picture?



What is the difference between area and perimeter?

THINK-PAIR-SHARE

King Arthur is having a Grand Feast at the end of one of his battles and each student in your class is invited. **Suggest** the **dimensions** of the table that could seat you all and fit a banquet. What assumption must you make before you begin your calculations?

Discuss your thoughts with a partner and then share with the rest of the class.

Perimeter is the distance **around** a two-dimensional shape (as shown in the examples; see also page 48).





Area is the amount of space **inside** a two-dimensional shape, as shown here.



Select a picture for each term, perimeter and area, or come up with your own, which you will always associate with the appropriate term so you will never mix them up!



■ What does area mean to this person?

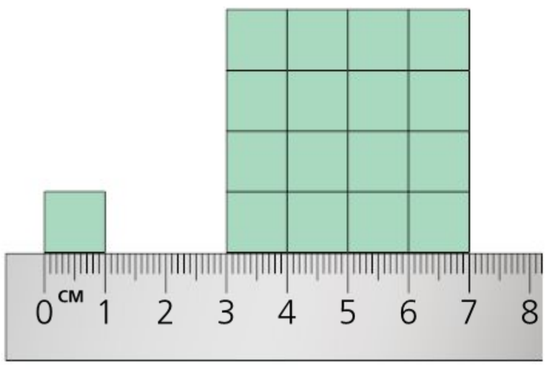
WHAT MAKES YOU SAY THAT?

What's going on here? Do we need think about perimeter or area in each case? Why?



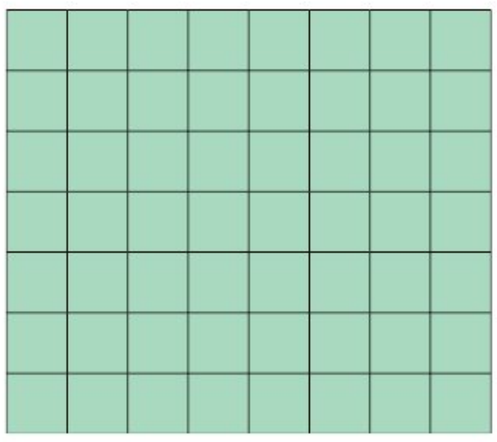
WHY IS AREA MEASURED IN UNITS SQUARED?

This diagram shows some examples of squares.



The square on the left is a centimetre squared – it is a special square that measures 1 cm × 1 cm. When we say 'an area of 12 cm^2' we mean that you can fit 12 of these one-by-one squares inside it, 12 squares that are each 1 cm × 1 cm.

Try this. How many squares are there in the image below? How can you **solve** this without counting them all?

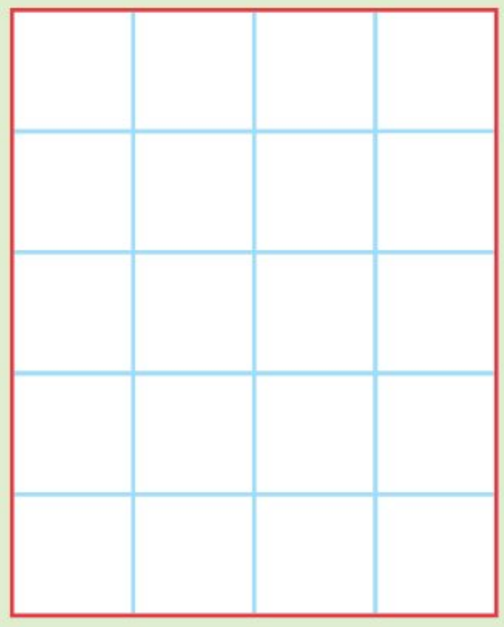


$_{rectangle}$ = ×

If we measured around the object above, what would we get as the perimeter? Is there a faster way to find it?

$_{any\ polygon}$ = sum of all side lengths

Example



Determine the area and perimeter of this rectangle drawn on centimetre squared paper.

Solution

We may count all the squares within the rectangle to determine its area, or we can simply multiply the number of squares in a row by the number of squares in a column.

$_{rectangle}$ = ×

$_{rectangle}$ = 4 cm × 5 cm

$_{rectangle}$ = 20 cm^2

Since the sides of each square on the grid are 1 cm, we can conclude that the top and base of the rectangle each measure 4 cm, and the two sides each measure 5 cm. Thus:

= sum of all side lengths

= 4 cm + 5 cm + 4 cm + 5 cm

or

= 2(4 cm) + 2(5 cm)

= 18 cm

Example



Determine the area and perimeter of the cinema screen.

Solution

Before we begin, we need to ensure we are using the **same units** for all dimensions.
Let's convert all dimensions to metres to avoid dealing with excessively large numbers.

Now we can **calculate** the area and perimeter.

$_{rectangle}$ = ×

$_{rectangle}$ = 30.0 m × 16.4592 m

$_{rectangle}$ = 493.776 m^2

and

= sum of all side lengths

= 30.0 m + 16.4592 m + 30.0 m + 16.4592 m

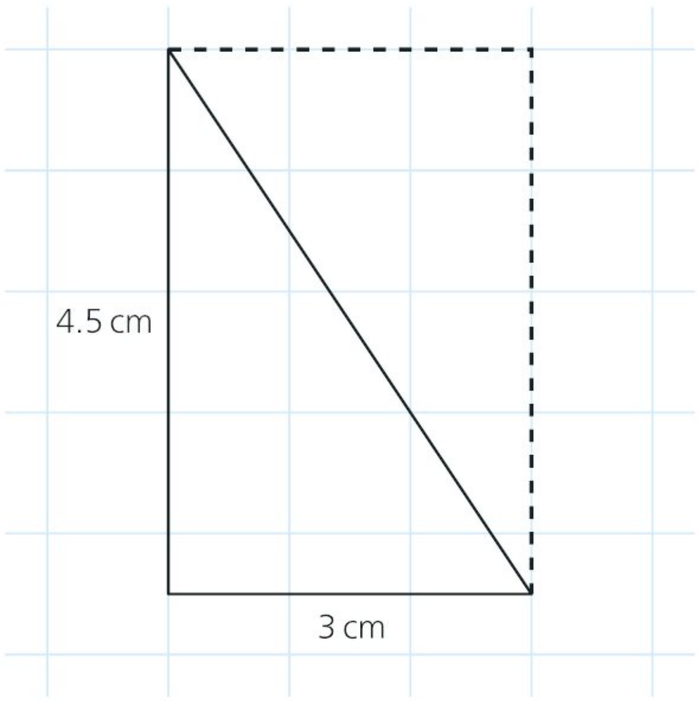
= 92.9184 m

ACTIVITY: Area of a triangle

ATL

- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions
- Creative-thinking skills: Use brainstorming and visual diagrams to generate new ideas and inquiries; Apply existing knowledge to generate new ideas, products or processes

1. Draw a square and **determine** its area.
2. Draw a line across the diagonal of the square and shade in one half of the square.
3. What fraction of the square does this area represent?
4. Can you come up with a formula or general rule for the area of a triangle?
5. **Determine** the area of the shaded figure shown here.



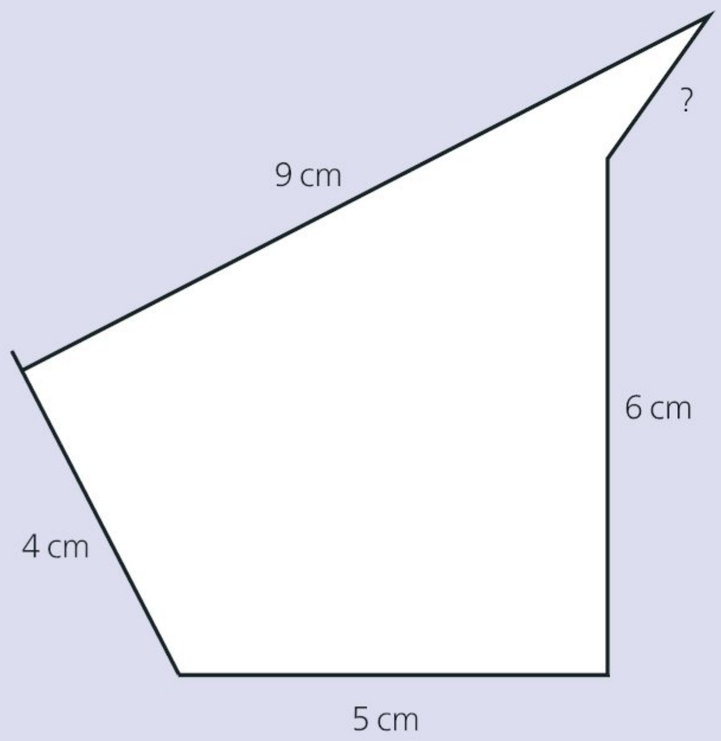
Assessment opportunities

In this activity you have practised skills that are assessed using Criterion B: Investigating patterns.

Example

How does knowing the perimeter help us determine a missing side length?

Determine the missing side length of this shape if the perimeter is 28 cm.



Solution

We know that the perimeter is 28 cm. This means adding all the sides together – including the unknown (?) – adds up to 28. Therefore we can write:

$$5\,\text{cm} + 4\,\text{cm} + 9\,\text{cm} + 6\,\text{cm} + ? = 28\,\text{cm}$$

because adding the sides gives the perimeter, and we are told the perimeter is 28 cm.

Hint

Have you forgotten how to work backwards to find an unknown? Flip back to Chapters 2 and 4 to refresh your algebra.

It is simpler to simplify first!

$$24\,\text{cm} + ? = 28\,\text{cm}$$

Then isolate the unknown, ?

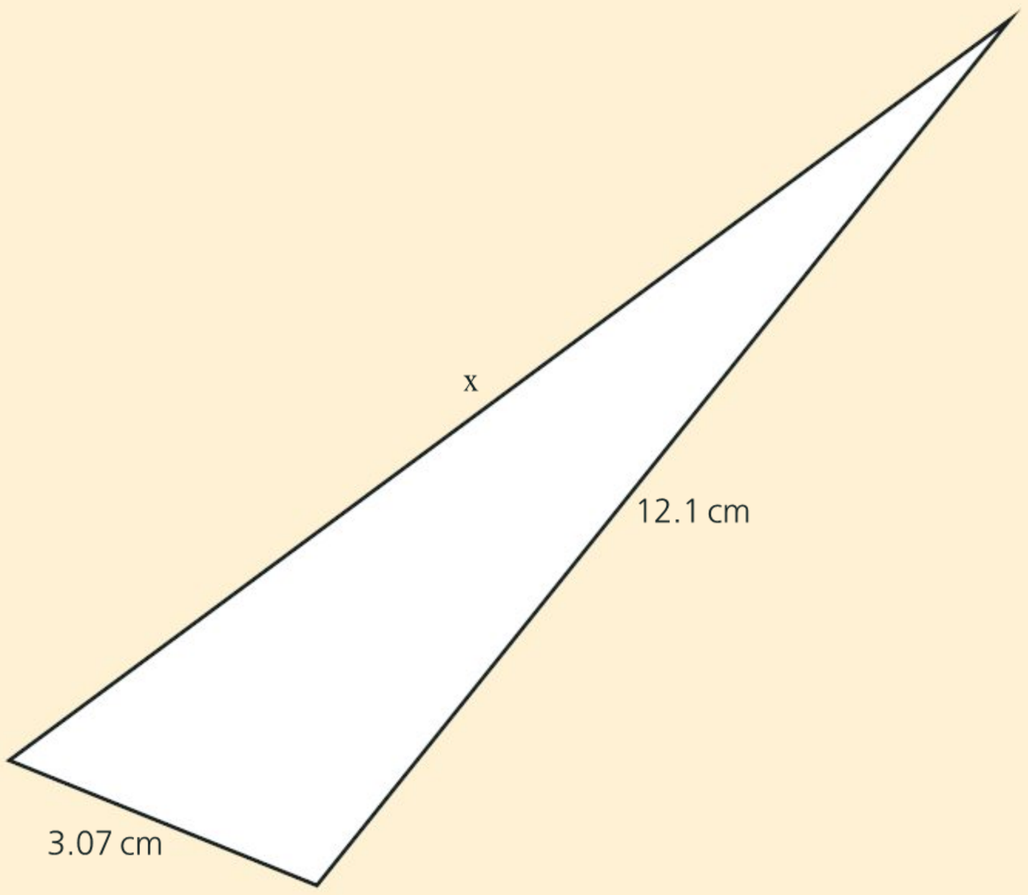
$$? = 28\,\text{cm} - 24\,\text{cm}$$

$$? = 4\,\text{cm}$$

So the missing side length in the figure is 4 cm.

Example

Now try this one. Determine the missing side length, x, if the perimeter is 28 cm.



Solution

How does knowing the perimeter help us determine the missing side length?

We know that the perimeter is 28 cm, which means adding all the sides together – including the unknown (x) – sums up to 28. Therefore we can write:

$$3.07\,\text{cm} + 12.1\,\text{cm} + x = 28\,\text{cm}$$

because adding the sides gives the perimeter, and we are told the perimeter is 28 cm.

It is simpler to simplify first, so

$$15.17\,\text{cm} + x = 28\,\text{cm}$$

Then isolate x

$$x = 28\,\text{cm} - 15.17\,\text{cm}$$

$$x = 12.83\,\text{cm}$$

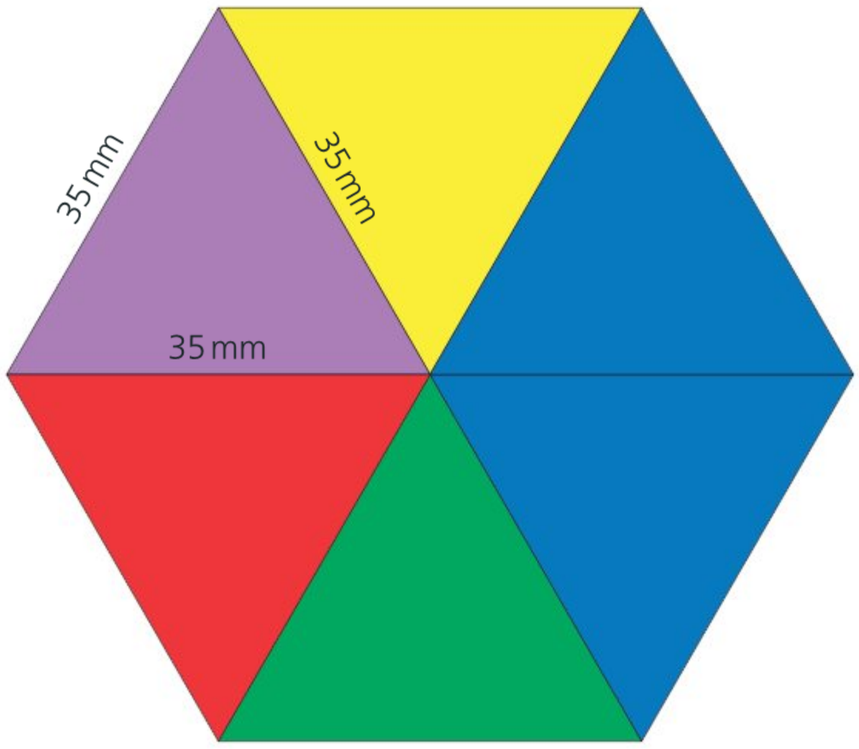
So the missing side length in the figure is 12.83 cm.

ACTIVITY: In a spin

ATL

- Communication skills: Make inferences and draw conclusions
- Critical-thinking skills: Draw reasonable conclusions and generalizations
- Creative-thinking skills: Apply existing knowledge to generate new ideas, products or processes

If the area of the green triangle is 530.44 mm^2, what is the area of the entire regular hexagon?



Is this a fair spinner? Why or why not? **Use** mathematical language in your response.

Can you come up with a shortcut way to **determine** the perimeter of this figure? Of all regular polygons? **Justify** your response.

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

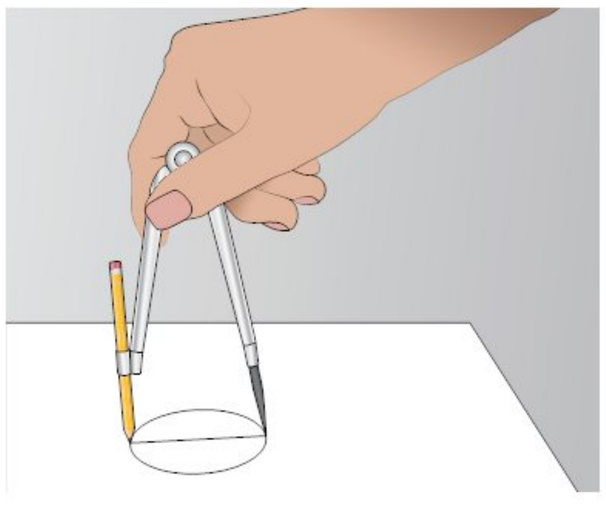
How are circles different from polygons?

ACTIVITY: A roundabout way to calculate circumference

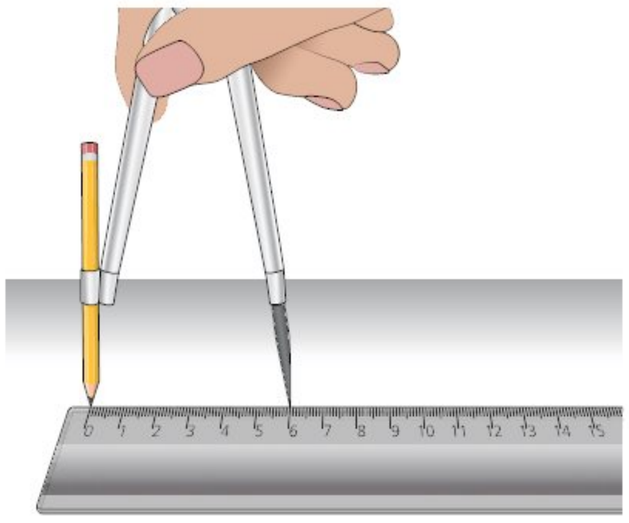
ATL

- Communication skills: Make inferences and draw conclusions; Use and interpret a range of discipline-specific terms and symbols
- Critical-thinking skills: Draw reasonable conclusions and generalizations
- Creative-thinking skills: Use brainstorming and visual diagrams to generate new ideas and inquiries; Apply existing knowledge to generate new ideas, products or processes

1 **Draw a circle of any size using your compass. If you are unsure how to do this, use this search term: 'How to draw a circle using a compass'.**
2 **Draw a line from one side of the circle to the other, ensuring it crosses the centre of the circle. This is called the diameter.**
3 **Measure and record the diameter using your compass and a ruler.**



Using a compass to determine diameter



Measuring space between compass needle and pencil

4 **There are no straight edges in a circle, so how will you measure the perimeter? Take a piece of string and trace it around the circle. Cut off any excess string, then measure the length of this string. This is the distance around your circle – called the circumference of the circle.**
5 **Repeat these steps for two more circles, each of different size.**
6 **For each circle, calculate its diameter divided by its circumference. What do you notice?**
7 **Share your findings with a partner and discuss any commonalities (things you have both found out). What are the possible human errors?**

Assessment opportunities

In this activity you have practised skills that are assessed using Criterion B: Investigating patterns and Criterion C: Communicating.

'Measure the middle and around
Divide so a number can be found
Every circle, great and small –
The number is the same for all'

Cindy Neuschwander, from Sir Cumference and the Dragon of Pi

! Take action

! Read the book *Sir Cumference and the Dragon of Pi* and then write your own story about Sir Cumference, Lady Di of Ameter and the Dragon of Pi to help inform others about measuring circles.

EXTENSION

In this chapter, we are using 'circumference' to refer to the distance around a circle. What other term have we used earlier in this chapter when describing distances around shapes? Can we **apply** that term to circles? Why or why not?

Links to: Visual arts



Circle of circles, Wassily Kandinsky

Wassily Kandinsky (1866–1944) was one of the first abstract artists. A painter and art theorist from Russia, he explored geometrical elements in his works and published *Point and Line to Plane* in the process. Here, he suggests that the circle is 'the most peaceful shape and represents the human soul'.

When carrying out the circumference activity, you may have noticed that dividing the diameter by the circumference of any circle, regardless of its size, always results in a very similar number. Due to measurement errors, you may not have calculated the exact same number every time, but were you to use precise instruments to perform your measurements, you would have come up with 3.1415, **every single time**. In fact, the number is an irrational number whose digits go on forever without pattern – but it is always exactly the same, whether you are measuring a circle that is the **cross-section** of a molecule, or the cross-section of a planet. We call this very special number pi (π).

We can never know the accurate circumference or area of a circle, because we don't know the true value of pi. To be as accurate as we can, we always use the pi button on our calculators rather than using a rounded number like 3.14.

Example

Calculate the circumference of a circle in Kandinsky's 'Circles in a circle' with diameter 88 cm.

Solution

Use the equation we derived in the last activity:

$\pi = -$

Substitute the number we have been given:

$\frac{\pi}{1} = \frac{}{88}$

And **solve** for :

$= 88\pi\,\text{cm} = 276.46\,\text{cm}$

What parts of the formula did you need to multiply to come up with the circumference? Can you come up with a formula for circumference of a circle?

Example

Calculate the circumference of a circle with **radius** 7 mm.

Solution

Since the radius only goes halfway across the circle, double it to get the diameter.

So

$= 2 \times 7\,\text{mm} = 14\,\text{mm}$

If

$= \pi$

Then

$= 14\pi = 43.96\,\text{mm}$

AREA OF A CIRCLE

For decades, farmers in the UK reported mysterious appearances of very large circles in their crops. The public often attributed these circles to UFO landings or beams. In 1992, two teenagers in Hungary created a crop circle with a 36 m diameter as a hoax, and were sued for the lost profits of wheat within the circle. How much wheat (in metres squared) was lost?

Here we are concerned with the **area** of the circle – we are looking at the amount of wheat **inside** the circle. First, let's look at the area of a circle and then we can apply it to our problem.

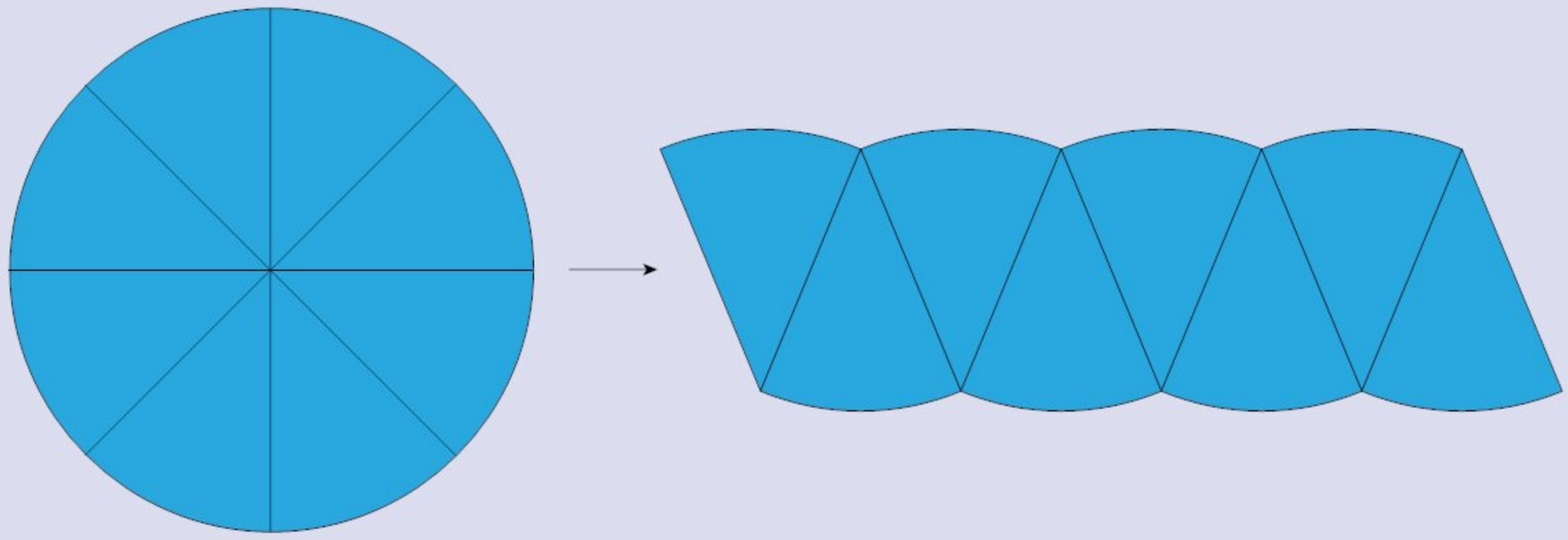


Example

1 Draw a circle on a blank sheet of paper. **Measure** the radius and **calculate** the circumference of your circle. Let's calculate what half the circumference would be too, just for fun.

2 Divide that circle into as many equal wedge-shaped pieces as you can, and cut out the pieces.

Fit the pieces into one another, as shown in the diagram.

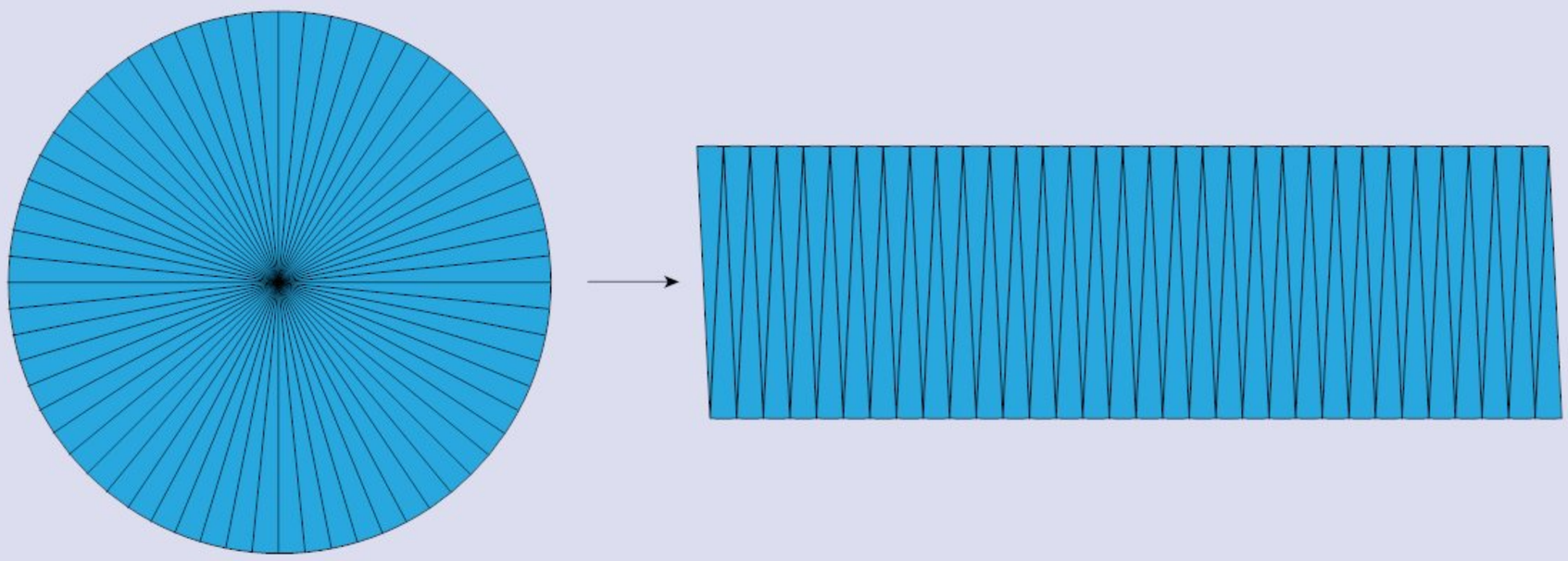


3 Watch how to do this at: www.youtube.com/watch?v=YokKp3pwVFco

4 Notice that you've made something like a parallelogram. If you made your wedges very narrow, how would your shape change?

5 **Measure** the lengths of the sides of your parallelogram. Are they close to anything you measured or calculated earlier? Why do you think your measurements aren't **exactly** the same?

6 If you were to try this with extremely small wedges, your rectangle would look like this.



7 How would you **calculate** the area of this rectangle? The link above explains how the area of the rectangle is related to the area of the circle from which it was made.

$= \times \frac{1}{2}$

But we know that $C = 2\pi$, so let's substitute that into the equation for area:

$= \times \frac{1}{2}(2\pi)$

And now simplify, as we know that $2 \times \frac{1}{2} = 1$, so these cancel each other out leaving:

$= \pi$

8 Collect the like terms:

$= \pi^2$

If the area of a rectangle made up of circle wedges is π^2, then the area of the circle is also:

$= \pi^2$

Let's look at that crop circle again. If $= \pi^2$, and $= 36\,\text{m}$, then:

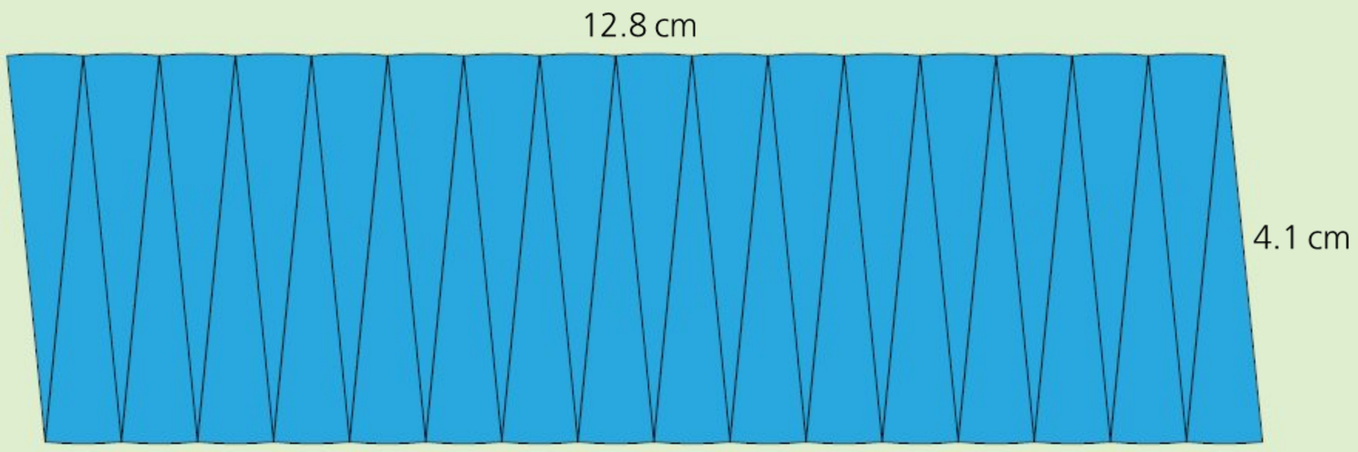
$= \pi(36)^2$

$= \pi 1296$

$= 4072.50\,\text{m}^2$

Example

The almost-rectangle below has a height of 4.10 cm and a width of 12.88 cm. Using **two** different methods, **determine** the area of the circle that would result when joining the wedges back together. Be sure to use the π button on your calculator and round your answer to the nearest hundredth!



Solution

Method 1: The area of the rectangle is equal to the area of the circle. So, we can use the formula for finding the area of a rectangle.

area of a rectangle = base × height

$= 4.10\,\text{cm} \times 12.88\,\text{cm}$

$= 52.81\,\text{cm}^2$

Method 2: We also know the height of the rectangle is equal to the radius of the circle, so we can use the formula for area of a circle.

$= \pi r^2$

$= \pi(4.10)^2$

$= 52.81\,\text{cm}^2$

Note: only rectangles that are made from a circle can have the same area as a circle. In these circles the radius of the circle is the same as the height of the rectangle.

ACTIVITY: Land grab

ATL

- Collaboration skills: Build consensus; Listen actively to other perspectives and ideas
- Communication skills: Make inferences and draw conclusions; Use and interpret a range of discipline-specific terms and symbols
- Critical-thinking skills: Draw reasonable conclusions and generalizations; Test generalizations and conclusions
- Creative-thinking skills: Use brainstorming and visual diagrams to generate new ideas and inquiries; Apply existing knowledge to generate new ideas, products or processes

As a gesture of goodwill, a sultan awards 1000 m of stone wall fencing to a villager who saved his child. He tells the villager he may own whatever land he can enclose in the stone wall. The villager is clever, and knows that different shapes with the same perimeter can have different areas. He hires you and a partner to figure out the shape that would yield the greatest area. What shape can you come up with? **Justify** your response.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding and Criterion B: Investigating patterns.

Can everything be broken down into a few basic shapes?

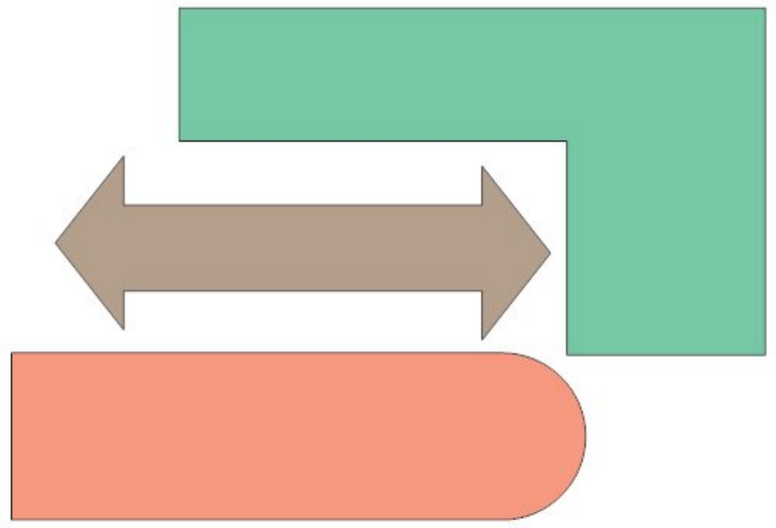
Creative thinking: creating novel ideas and considering new perspectives

In the example below, we examine one way to divide up a shape. Is there another way? If so, does it yield the same answer, and is it more or less efficient? **Justify** your response.

HOW MANY SHAPES DO YOU SEE?

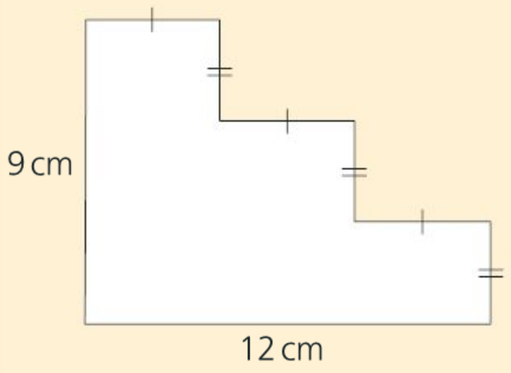
Is each diagram just one shape, or can you find smaller shapes within? Try to divide figures into sub-shapes by drawing lines.

All the shapes to the right are classified as '**composite figures**'. Composite literally means 'placed together'. Any figure made up of two or more shapes that have been 'placed together' is a composite figure. How might we calculate the area of a composite figure?



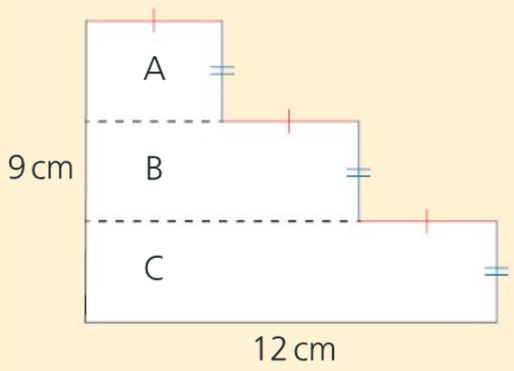
Example

Calculate the area and perimeter of the composite figure in the diagram:



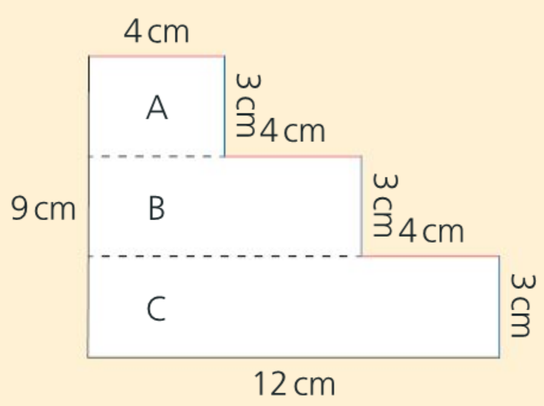
Solution

Step 1: Divide the shape into basic polygons.



Step 2: Work out the missing measurements. If we were to connect the three red lines, they would be as long as the base of the figure. That means that they ought to add to 12 cm. Since they each have a single tick through them, we know they are three equal lengths. How can we **determine** the length of each red line?

Do the same for the lines with the double ticks. The three equal blue lengths add to 9 cm.



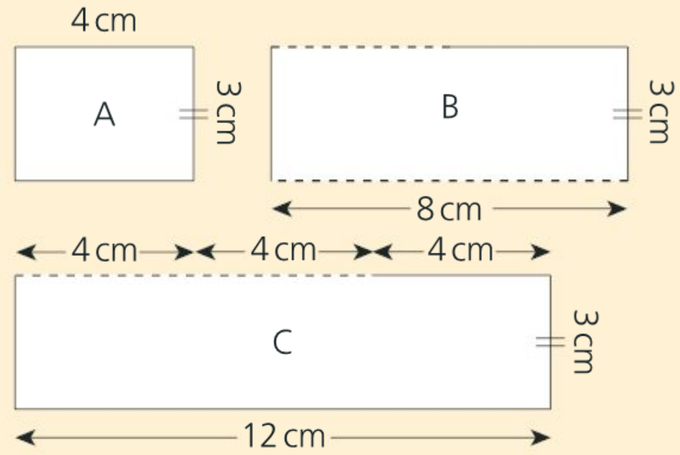
Now we have a completely labelled diagram, so we can determine the perimeter:

$4\,\text{cm} + 3\,\text{cm} + 4\,\text{cm} + 3\,\text{cm} + 4\,\text{cm} + 3\,\text{cm} + 12\,\text{cm} + 9\,\text{cm} = 42\,\text{cm}$

… or, more simply – the three red lines are each 4 cm long, and the three blue lines are 3 cm each, so:

$(3 \times 4\,\text{cm}) + (3 \times 3\,\text{cm}) + 12\,\text{cm} + 9\,\text{cm} = 42\,\text{cm}$

Step 3: Now that we have three individual rectangles, we can calculate the area of each, and then add them up.



Rectangle A:
$= 4\,\text{cm} \times 3\,\text{cm} = 12\,\text{cm}^2$

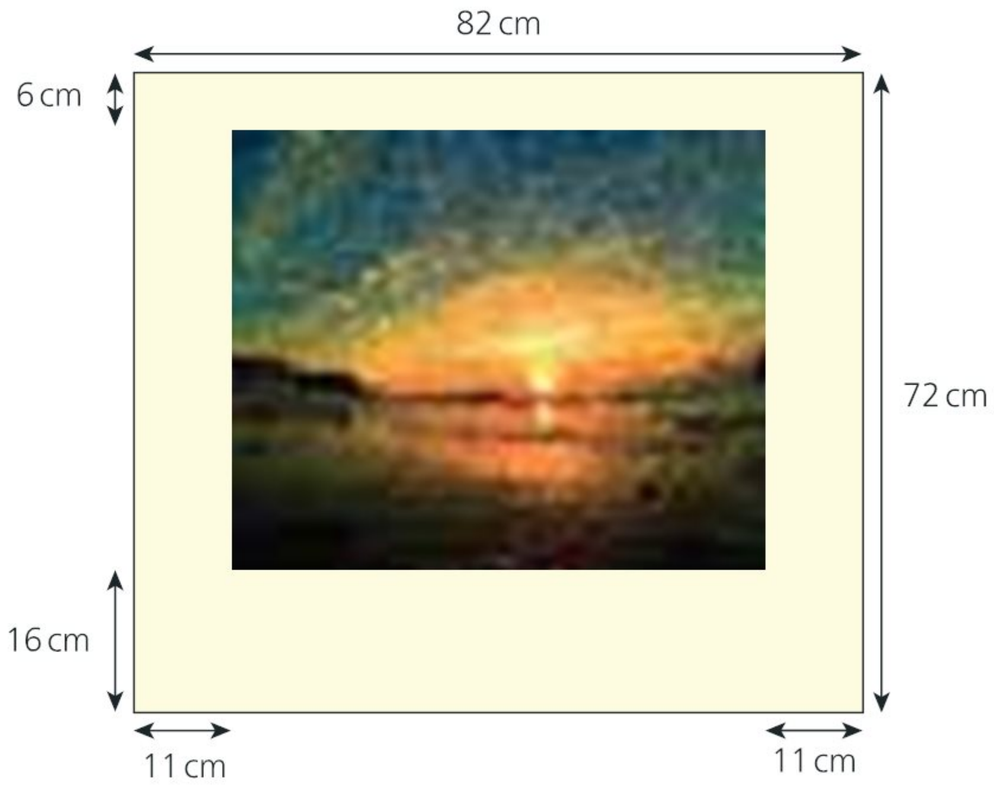
Rectangle B:
$= 8\,\text{cm} \times 3\,\text{cm} = 24\,\text{cm}^2$

Rectangle C:
$= 12\,\text{cm} \times 3\,\text{cm} = 36\,\text{cm}^2$

Total area: $12\,\text{cm}^2 + 24\,\text{cm}^2 + 36\,\text{cm}^2 = 72\,\text{cm}^2$

EXTENSION

Composite figures can also be made from shapes that have had other shapes **removed**. Calculate the area of the **frame** for this picture.



ACTIVITY: Composite figure art

ATL

- Organization skills: Use appropriate strategies for organizing complex information

Cut various shapes from three or more differently coloured pieces of construction paper, or card, then create an artwork collage by gluing them onto a single page. If you prefer, you could simply **draw** a sketch using coloured pencils. **Calculate** the total area each colour takes up.

If you needed to magnify your art 30 times to be on a wall in your school hallway, and you needed two coats of paint for each colour, what would be your total paint cost? Assume it takes 1 L of paint to cover 1000 cm^2.

- **Small tube: £3.89 for 600 mL**
- **Medium can: £10.80 for 1 L**
- **Large can: £12.50 for 2.5 L**

Assessment opportunities

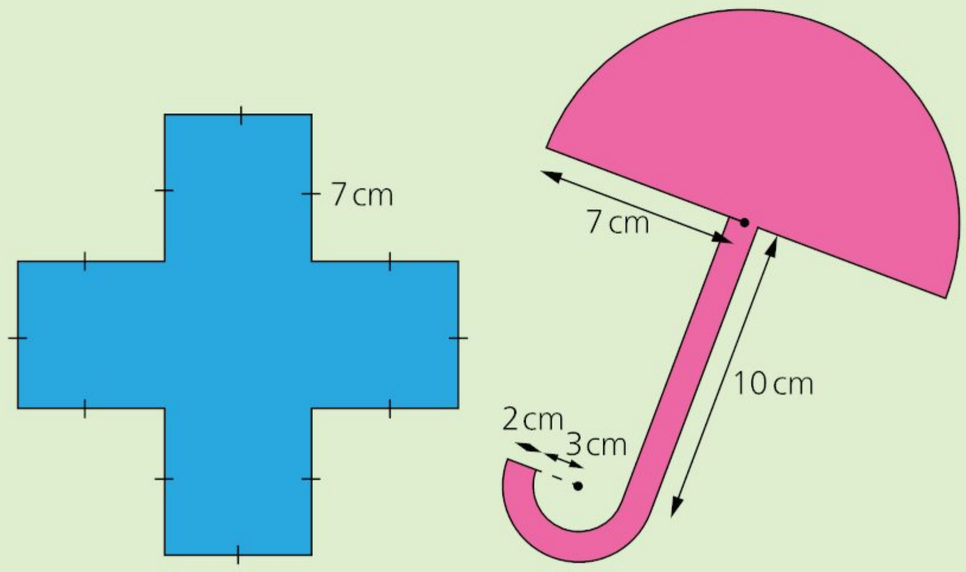
This activity can be assessed using Criterion D: Applying mathematics in real-life contexts.

Links to: Visual arts

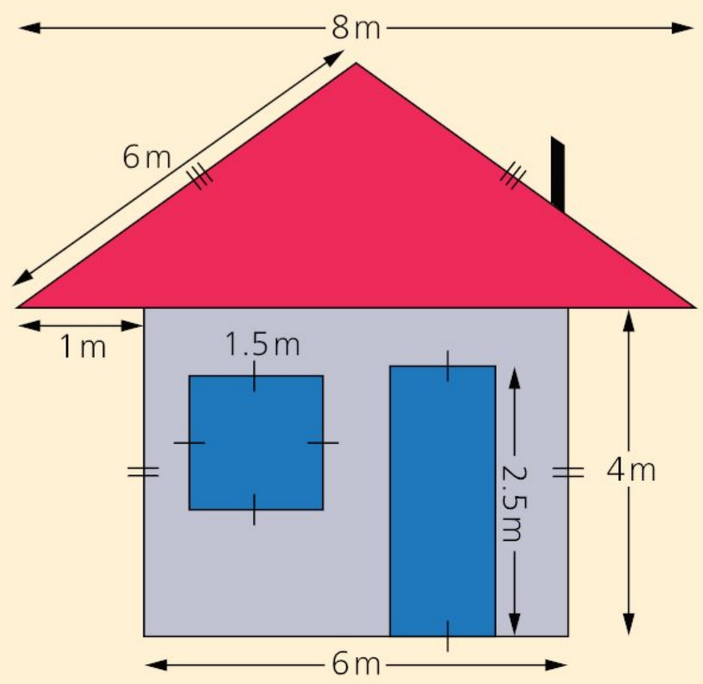
Impressionism is characterized by elements such as visible brush strokes, far less emphasis on detail and more emphasis on light and movement. Can you identify the two Impressionists whose styles have been combined to form the painting in the Extension box?

PRACTICE EXERCISE

1 **Draw** dotted lines in the images below to break them up into regular shapes.



2 Determine the area and perimeter of the figures above.



3 If a can of paint covers 3500 cm^2 then how many cans would be needed to paint **two coats** of the grey front of the tool shed in the figure?

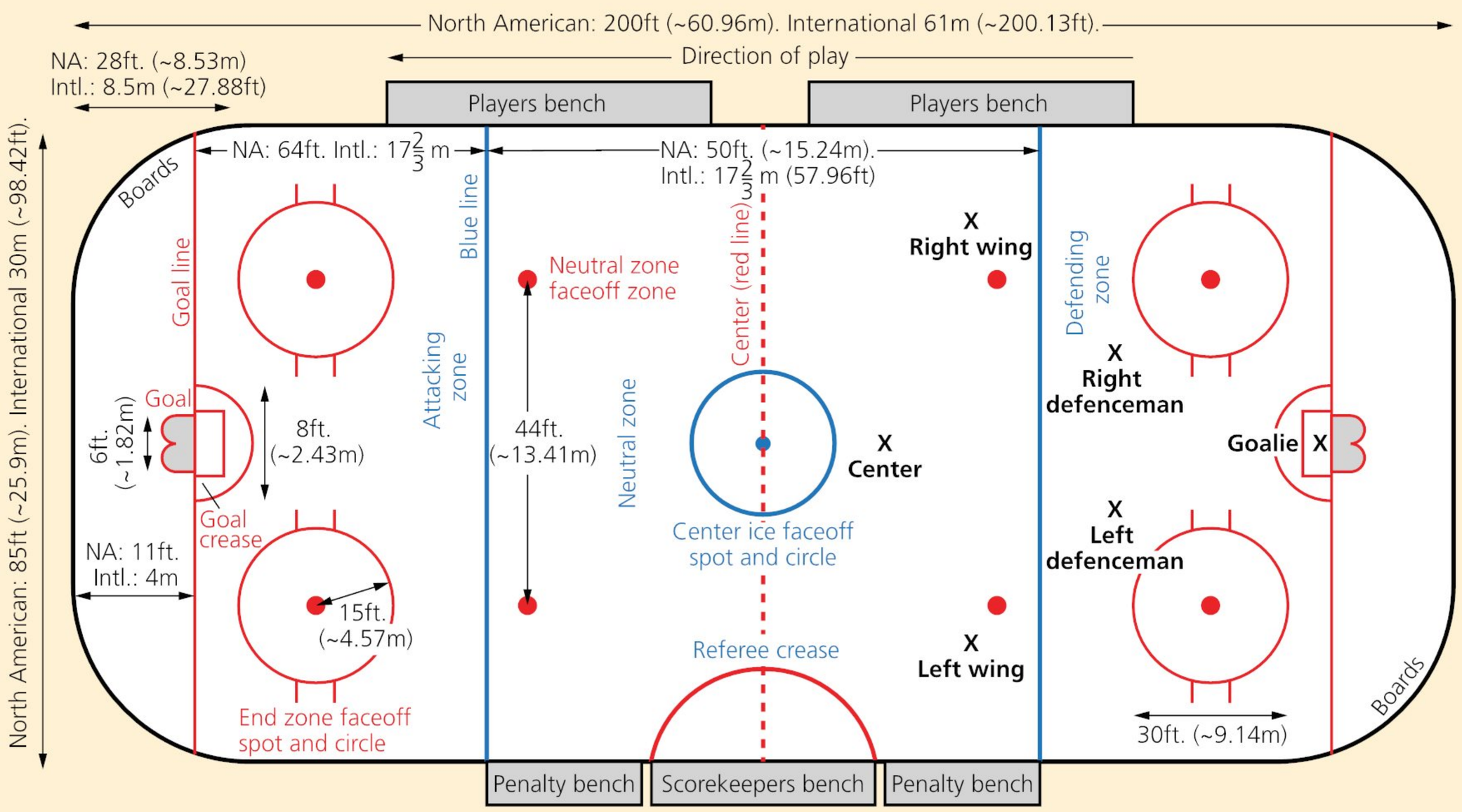
Hint

Can you buy a fraction of a can of paint?

4 **Estimate** the area of an Olympic ice rink, based on the diagram below. **Justify** your response.

Hint

Make sure that you estimate the area using dimensions in similar units.



SUMMATIVE ASSESSMENT: Modern-day Rome

ATL

- Creative-thinking skills: Generating novel ideas and considering new perspectives
- Transfer skills: Using skills and knowledge in multiple contexts

Public baths in ancient Rome were known for intricate tile work. Opposite are some of the many mosaics that have been found in the ruins of Pompei and Caracalla. The mosaics are made from tiny stones that are all roughly the same size, ordered into decorative slabs. The slab motifs were commonly repeated to cover large areas.

Suppose you are hired to lay the tiles for a modern-day swimming pool that replicates a Roman bath. This means creating a decorative frame around the outside edge of the pool made up of large 0.5 m by 0.5 m slabs, each containing the same mosaic motif.

Your advantage over the Romans is that you do not need to find little rocks of roughly the same size to create your mosaic – you have mini tiles, each 5 cm^2. Specifically, you have:

- **12 450 × 5 cm^2 white mini tiles**
- **10 970 × 5 cm^2 black mini tiles**
- **8180 × 5 cm^2 brown mini tiles.**

The pool measures 54 m × 24 m. How many 0.5 m^2 slabs will you need to frame the pool?

Design a mosaic whose motif will, when repeated around the pool, **use** up all of your mini tiles. **Draw** the slab to scale and provide its dimensions. Comment on how the measurements have affected your design. Is it possible to have pools with different dimensions that would use up the same number of slabs? **Justify** your response.

Assessment opportunities

This activity can be assessed using Criterion D: Applying mathematics in real-life contexts.







■ Tiles still intact at ancient Roman baths

Reflection

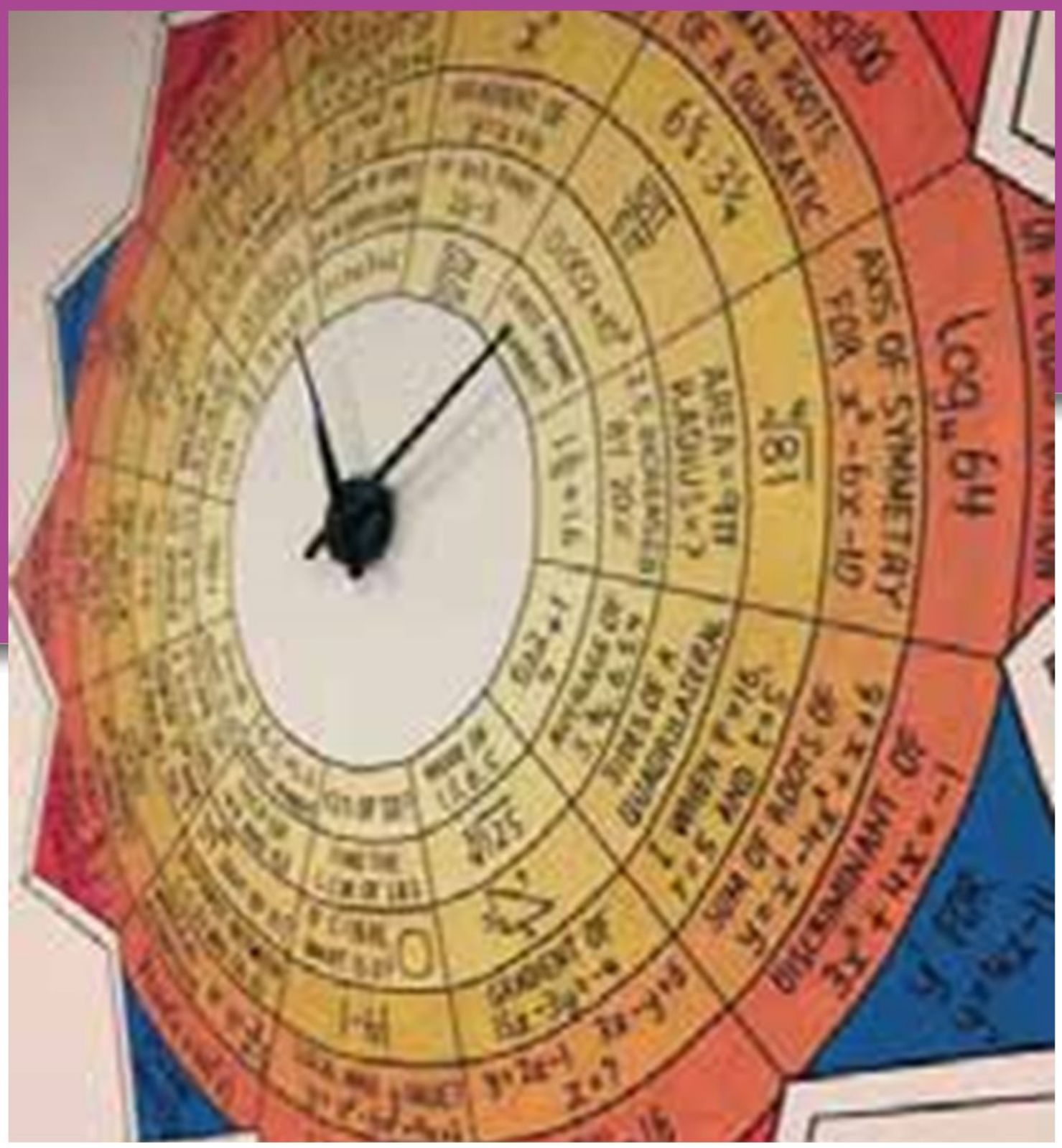
Consider these end-of-chapter reflection questions:

- Why do we consider ourselves 'civilized'?
- Does mathematics play a role in those things we define as 'civilized'?
- Can we say that others are not civilized or have not been in the past?
- How do we 'measure' civilization?
- Has this chapter changed what you understand by civilization?

Use this table to reflect on your own learning in this chapter.						
Questions we asked	Answers we found	Any further questions now?				
Factual: How can we represent size on paper? Are all triangles created equal? What is the difference between area and perimeter?						
Conceptual: Can size be represented on paper? What difference does a degree make? How can you describe a shape? How are circles different from polygons?						
Debatable: How do we define 'big'? Is it important to have a common language to communicate quantity? Can everything be broken down into a few basic shapes?						
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?				
		Novice	Learner	Practitioner	Expert	
Communication skills						
Creative-thinking skills						
Critical-thinking skills						
Transfer skills						
Collaboration skills						
Learner profile attribute(s)	How did you demonstrate your skills as a risk-taker in this chapter?					
Risk-taker						

6 What's next?

Mathematical **logic** helps us to find **general rules** in **quantities** and relationships and to make **exciting, innovative discoveries**.



CONSIDER THESE QUESTIONS:

Factual: How does the order change the answer? How do magic squares work? How can we name and identify shapes all around us? How can we place numbers and things into a Venn diagram?

Conceptual: How and when are mathematical innovations made? How good is your estimation? Why simplify? How can we group numbers together based on their quantities (in dots)? How can you represent your learning creatively?

Debatable: Is there ever a 'best' method? Can mathematics make the perfect picture? How much can we rely on models?

Now **share and compare** your thoughts and ideas with your partner, or with the whole class.

PRIOR KNOWLEDGE

Reflect on what you already know about:

- how to multiply single digit numbers
- the order for more than one operation in an expression
- how to use a ruler to divide a picture into sections.

IN THIS CHAPTER, WE WILL …

- **Find out** how far we have come this year and how much we have learned.
- **Explore** the ideas from previous chapters in new ways, each linked to a related concept.
- **Take action** by creating interesting and innovative displays for our classroom and school, which will contribute to the wider school appreciation of mathematics.

These Approaches to Learning (ATL) skills will be useful …

- Communication skills
- Creative-thinking skills
- Media-literacy skills
- Critical-thinking skills
- Collaboration skills
- Transfer skills

We will reflect on this learner profile attribute …

- **Risk-taker** – We approach uncertainty with forethought and determination. We work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

KEY WORDS

convert
factor
hexagonal
horizontal
order
pentagonal
vertical

Assessment opportunities in this chapter:

- **Criterion A**: Knowing and understanding
- **Criterion C**: Communicating
- **Criterion D**: Applying mathematics in real-life contexts

MYP Mathematics is focussed on learning through concepts and contexts. A concept is a 'big idea' – a principle or a notion that is enduring or long lasting and should take you into lifelong adventures of learning.

You have been handling **mathematical concepts** all through this book and you will see them regularly throughout your next few years in MYP. You will use your **conceptual understanding** as you solve problems, analyze issues and evaluate decisions which can have an impact on yourself, your community and on the wider world.

Related concepts are ideas which are useful for exploring mathematics in greater detail. The graphic shows the 12 related concepts that we use in MYP.

DISCUSS

What have you learned this year? Have you seen all of the concepts below in real-world settings? Have you applied them all? Do you feel **more** or **less** comfortable with mathematics? What has changed? What has not changed?

Complete the following:

'I used to think … Now I think …'





How does the order change the answer?

Change: A variation in size, amount or behaviour.

In earlier chapters, we practised the **order of operations** – which operations to do first and which to do after that.

Can you remember what the letters BEDMAS stand for? Can you remember your mnemonic device to help you remember order of operations?

PRACTICE EXERCISE

Calculate:

1 $2 + 7 \times 3 - 1$

2 $8 \div 4 + 7 \times 11$

3 $22 + 4 \div (3 - 1)$

4 $72 - 1 + 11 - 12$

5 $3 + (7 - 2)^2 \div 5$

6 $3 + 7 - 2^2 \div 5$

7 $-4 \times -\frac{11}{1} + 4 \times 5 + (-1)^2$

How smart is your calculator? Does it understand the order?

Type all the questions above straight into your calculator.

ACTIVITY: Colour-coding

ATL

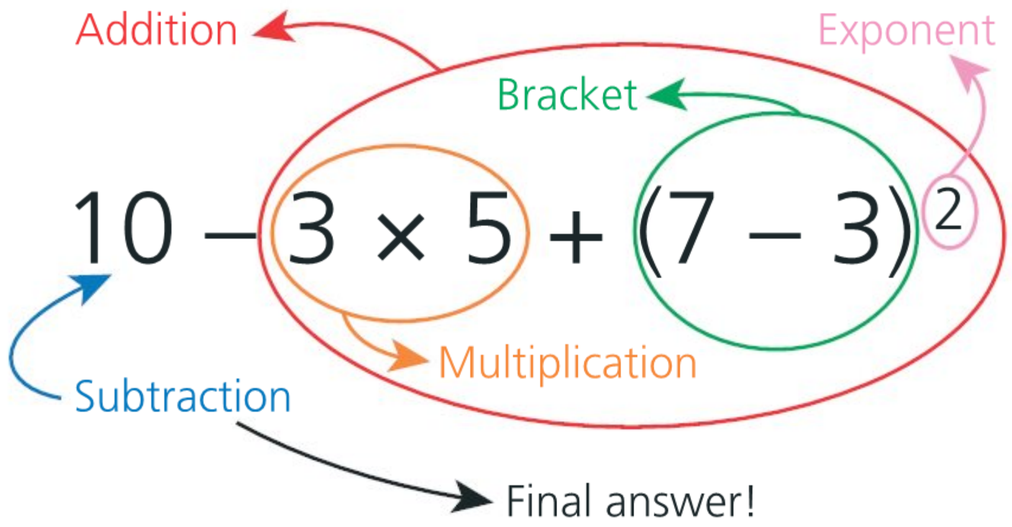
- Communication skills: Organize and depict information logically

Choose six colours and match each one to the operation listed below:

- **brackets**
- **exponents (powers)**
- **division**
- **multiplication**
- **addition**
- **subtraction.**

Example

$10 - 3 \times 5 + (7 - 3)^2$





Why simplify?

Using the colours you have chosen, now highlight each part of the questions you looked at before to identify which operation is which and to label the order they must be done in.

1 $2 + 7 \times 3 - 1$

2 $8 \div 4 + 7 \times 11$

3 $22 + 4 \div (3 - 1)$

4 $72 - 1 + 11 - 12$

5 $3 + (7 - 2)^2 \div 5$

6 $3 + 7 - 2^2 \div 5$

7 $-4 \times -\frac{11}{1} + 4 \times 5 + (-1)^2$

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion A: Knowing and understanding.

Simplification

The process of reducing to a less complicated form.



ACTIVITY: Cute cubes

ATL

- Information-literacy skills: Understand and use technology systems

Use your calculator to find:

642^3

and

641^3

Now subtract your answers from one another. What do you find?

Try putting the entire problem into the calculator at once:

$642^3 - 641^3 = ?$

Now try the same question in a different way

$642 \times 642 \times 642 - 641 \times 641 \times 641 = ?$

Do your answers match? What does this make you think about your calculations versus the calculator's calculations?

Assessment opportunities

- This activity can be used to assess Criterion B: Investigating patterns.

Is there ever a 'best' method?

There are lots of ways to multiply numbers. One way to multiply is by placing the two numbers we want to multiply together directly below one another.

METHOD 1: COLUMN METHOD

To multiply

73×65

place the numbers into columns, like this:

$$\begin{array}{r} 7\ 3 \\ \times\ 6\ 5 \\ \hline \end{array}$$

Next multiply the whole top number by the smallest number on the bottom:

$$\begin{array}{r} {}^{1}7\ 3 \\ \times\ 6\ 5 \\ \hline 3\ 6\ 5 \end{array}$$

Hint

Multiply the 3 X 5 first. This gives the answer 15 – write down the 5 and 'carry' the one with you for the next multiplication. Then multiply 5 X 7, which gives 35 (plus the carried 1 makes 36). Write this final answer in front of the 5.

So far, we have multiplied by the **5** in 6**5,** but what about the **6**0?

Equivalence

The state of being identically equal or interchangeable, applied to statements, quantities or expressions.

First place a zero in the right-hand column, so we can multiply by 6 instead of 60.

$$\begin{array}{r} {}^{1}7\ 3 \\ \times\ 6\ 5 \\ \hline 3\ 6\ 5 \\ +\ \ 0 \end{array}$$

Now repeat the same process as before, multiplying every term by 6 going from right to left:

$$\begin{array}{r} {}^{1}7\ 3 \\ \times\ 6\ 5 \\ \hline 3\ 6\ 5 \\ +4\ 3\ 8\ 0 \end{array}$$

Then add the two rows to give the final answer:

$$\begin{array}{r} {}^{1}7\ 3 \\ \times\ 6\ 5 \\ \hline 3\ 6\ 5 \\ +4\ 3\ 8\ 0 \\ \hline 4\ 7\ 4\ 5 \end{array}$$

PRACTICE EXERCISE

Using Method 1, multiply:

1	14×62	3	33×82
2	82×56	4	69×72

METHOD 2: THE BOX OR GRID METHOD

For this method, we draw a grid. Make sure you have the same numbers of rows and columns as digits in the question.

Here again, we will multiply

73 × 65

In this example, there are two digits in each of the numbers we wish to multiply, so our table will look like this:

Next break down the numbers 73 and 65 in the following way:

	60	5
70		
3		

Then multiply each side of the rectangle to fill in the empty boxes:

	60	5
70	4200	350
3	180	15

Now that we have all the multiples of the individual numbers, we add together all four boxes to get the final total:

	60	5
70	4200	350
3	180	15

4 2 0 0
3 5 0
1 8 0
1 5
4 7 4 5

PRACTICE EXERCISE

Using Method 2, multiply:

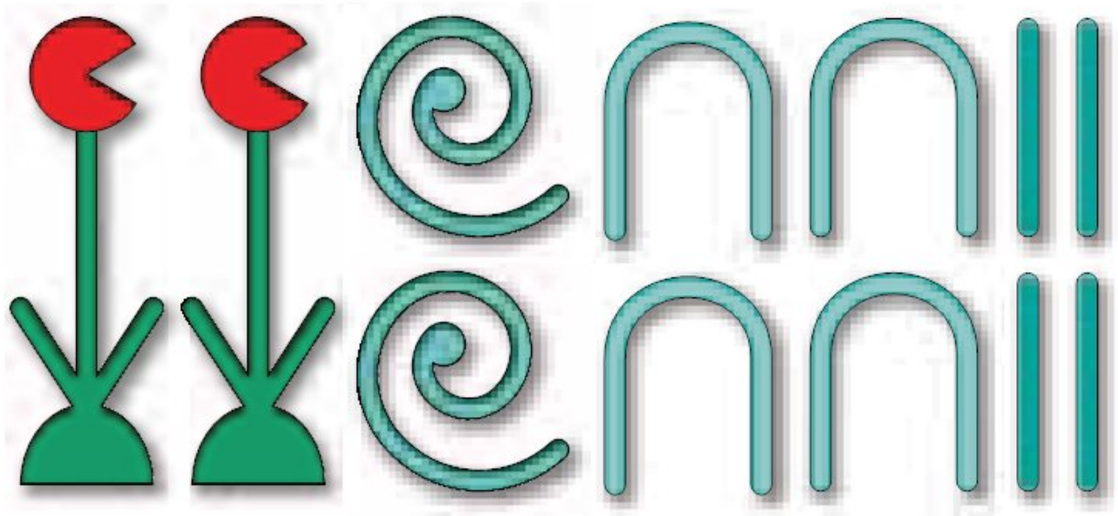
1	14 × 62	**3**	33 × 82
2	82 × 56	**4**	69 × 72

PRACTICE EXERCISE

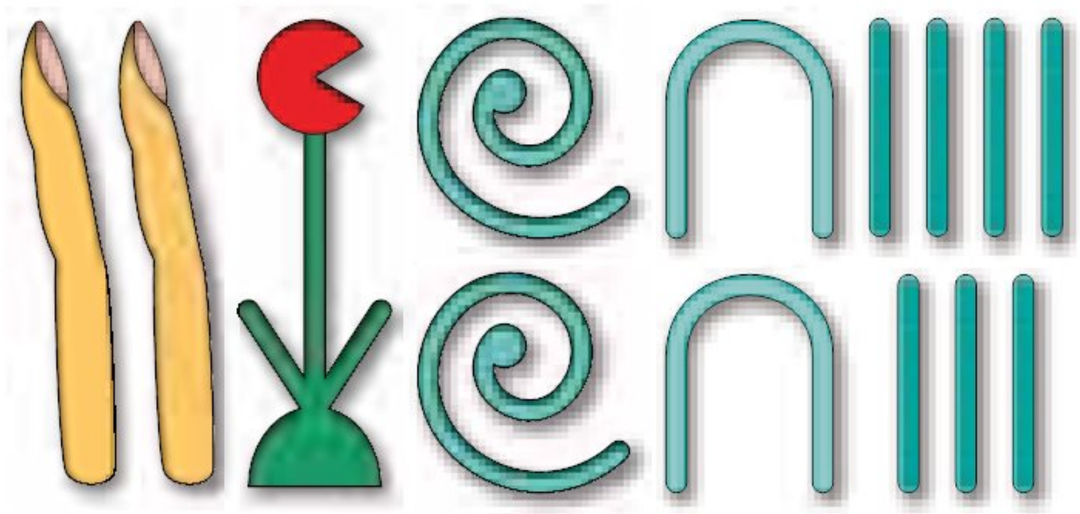
Using Method 3, multiply:

1	14 × 62	3	33 × 82
2	82 × 56	4	69 × 72

METHOD 3: COUNT LIKE AN EGYPTIAN



= 2244



= 21 227

To apply this method, you must be able to double numbers and then select a few to add at the end. That's it!

We are doing the same multiplication:

73×65

We want to find 73 'lots' of 65. Start with finding 1 lot of 65. Then double it, double it again and keep doubling until you approach (nearly get to) 73.

73	×	65

1	65
2	130
4	260
8	520
16	1040
32	2080
64	4160

Don't double it again because you will go over the desired number of 73!

Now we need to find a combination of the multiples which will give us a total of 73. Start with the biggest you can find and then the next biggest, until you have exactly the multiple you are looking for:

73	×	65

1	65
2	130
4	260
8	520
16	1040
32	2080
64	4160

$73 = 64 + 8 + 1$

Now look for the numbers resulting from the multiples of 64, 8 and 1 and add them together:

73	×	65

1	65
2	130
4	260
8	520
16	1040
32	2080
64	4160

$73 = 64 + 8 + 1$

$\therefore\ 73 \times 65 = 4160 + 520 + 65 = 4745$



METHOD 4: RUSSIAN PEASANT MULTIPLICATION

It is said that this method was used by people (usually the poorer, less educated people) to successfully multiply large numbers without having to do too much multiplication.

The steps for this method are as follows:

1. You must double the first term you wish to multiply and halve the second term.
2. If you have a remainder when you halve, ignore it. Drop it from the number entirely.
3. Once the second column is down to 1, stop! Then cross through any line which is even on the right-hand side.

Here we can see that the left-hand side has been doubled and the right-hand side halved until it reaches 1:

73	65
146	33
292	16
584	8
1168	4
2336	2
4672	1

double → left-hand side; halve → right-hand side

Now we simply add any remaining numbers on the left-hand side:

$$73 + 4672 = 4745$$

73	65
~~146~~	~~33~~
~~292~~	~~16~~
~~584~~	~~8~~
~~1168~~	~~4~~
~~2336~~	~~2~~
4672	1

double → left-hand side; halve → right-hand side

cross out any even numbers

This method is quite intriguing – why does it work? How does it work? Why can we ignore remainders? Why must we remove any even terms on the right-hand side? Consider how this method works in such an ingenious way.

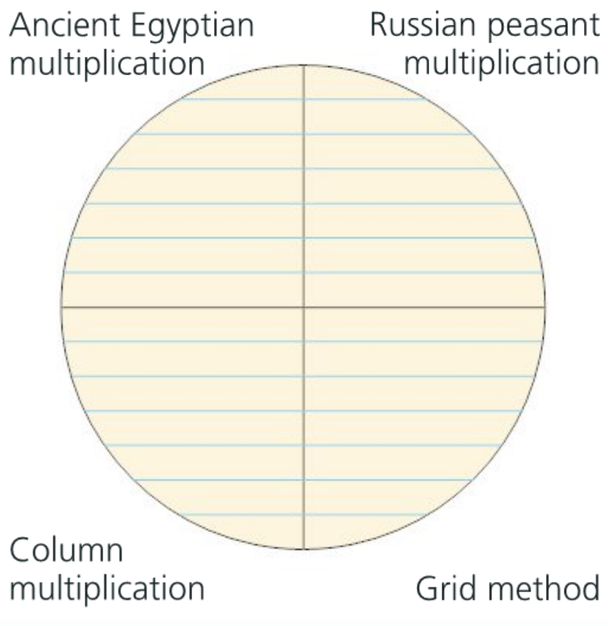
PRACTICE EXERCISE

Using any method you choose, multiply:

1	14×62	3	33×82
2	82×56	4	69×72

Circle of viewpoints

Consider the viewpoints of different learners who prefer one method over another. Copy the diagram below using a full page. In each segment put down the arguments you think a person choosing that method would say to defend their choice. When completed, shade in the segment or segments which reflect your own opinion.



How and when are mathematical innovations made?

Justification

Valid reasons or evidence used to support a statement.

ACTIVITY: Yes, but why?

ATL

- Media-literacy skills: Seek a range of perspectives from multiple and varied sources

What do you think was the world's greatest mathematical discovery or innovation … at that time or for all time?

Choose a topic, fact or process you have learned about this year and research who discovered it and when. Find out as much as you can about it. What led to the discovery? Did it involve a single person or did several civilisations know at similar times?

Here is the memorial to one such 'flash of genius'. Find out more about who Sir William Rowan Hamilton was, what he discovered and where.

Why did he carve his idea into a bridge? How did his breakthrough advance science and technology?



Assessment opportunities

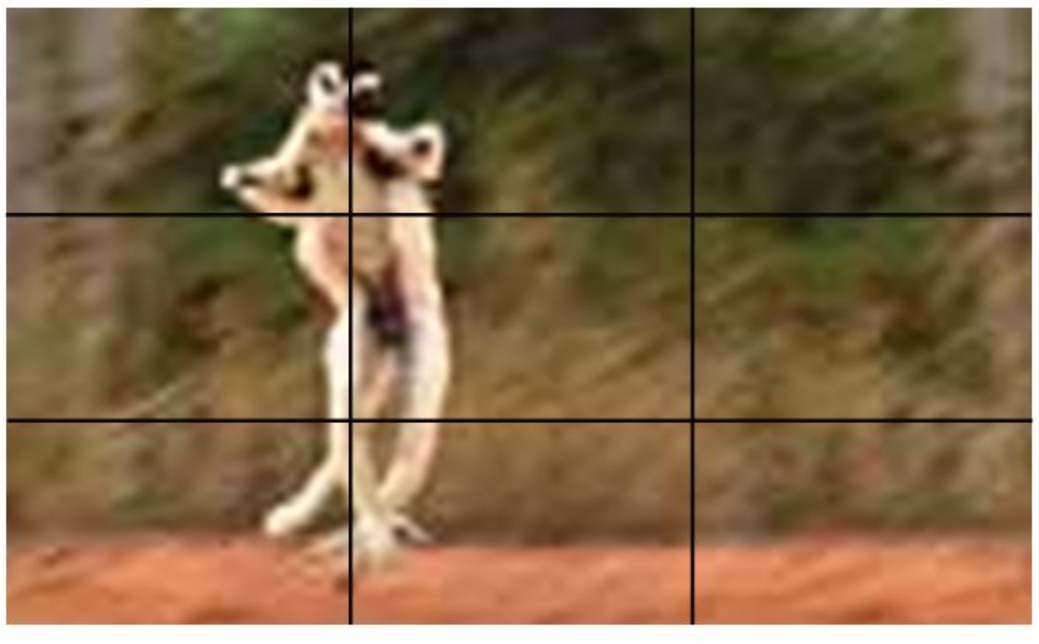
- In this activity you have practised skills that are assessed using Criterion C: Communicating.

CAN MATHEMATICS MAKE THE PERFECT PICTURE?

Generalization

A general statement made on the basis of specific examples.

Rule of Thirds



Have you ever taken a photo that's not quite right: a landscape that needs something to liven it up, a person who seems to be in the wrong place or who, somehow, is not the focal point? Your photo could be **unbalanced**.

One of the first concepts that student photographers learn about is the 'Rule of Thirds'. According to this rule, photographers should imagine their scene as a 3 by 3 grid. Placing the subject at an intersection of the grid, or along the grid lines, or within any three of the nine boxes makes for a well-balanced photograph.

Of course, rules are made to be broken and you can still take interesting photos that don't follow the Rule of Thirds. But you need to learn a rule before you can break it!

ACTIVITY: Mastering the rule

ATL

- Creative-thinking skills: Create original works and ideas; Use existing works and ideas in new ways

Analyze this picture.



1. Divide the image into thirds horizontally.
2. Divide the image into thirds vertically.
 - How many sections are there in the image now?
 - What fraction is each part?
 - Show this mathematically, as a product of fractions.
 - How many 'points of intersection' are in the image?
 - What is happening at each of these points of intersection?
 - How many fractions (or how much of the picture) is/are filled with water?
 - How many ninths of the picture are filled with sky?
 - Why do you think this rule makes a good photograph?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating and Criterion D: Applying mathematics in real-life contexts.

Links to: Language and Literature

If a picture is worth a thousand words, what does that mean?

Can you write a story inspired by a picture which follows this rule? Be creative. Could you reach a hundred words? A thousand?







Now let's combine both rules – the Rule of Thirds **and** the saying that a picture is worth a thousand words.

Write a story of 1000 words, placing an equal number of words in each box created by the Rule of Thirds. (If 1000 words is too long, try 100.)

How many words will you put in each section?

When you divide 1000 words by nine sections, there will always be a remainder. What creative ways can you find to handle the last word or any remaining words?

How good is your estimation?

Measurement is all around us. Often the measurements we use have been done for us but sometimes you must measure things for yourself, when you are cooking for example, or doing a scientific experiment.

> **Measurement**
>
> A method of determining quantity, capacity or dimension using a defined unit.

Other times you must **estimate** an amount without knowing exactly the quantities involved. Good spatial awareness and the ability to reason are helpful for this.

ACTIVITY: How good is your estimation?

ATL

- Collaboration skills: Exercise leadership and take on a variety of roles within groups

The hospitality industry is a name used for a category of work whereby people offer services to tourists, in hotels and on cruises for example.

In the American hospitality industry, there is an important rule called the 5/10 rule. This rule states that you should always acknowledge someone when you get to within 10 feet of them. This acknowledgement means that you should make eye contact or smile. At five feet apart, the employee must address the guest with a 'good morning' or similar greeting.

You should complete this activity in small groups. Nominate one person to act the part of the 'guest' and one person to be the 'employee', who must follow the 5/10 rule. Other members of the group will take accurate measurements of distance. You will need string and measuring devices such as a trundle wheel or measuring tape.

- **Each member of the group must cut off a length of string which they estimate to be 1 feet long.**
- **Measure the strings to see who has estimated the most accurate 1-foot length. (A foot is approximately 30.5 cm.)**
- **Find a corridor at least 20 feet long.**
- **The 'guest' and 'employee' should start walking from opposite ends of the 20-foot corridor and must stop when they think they are 10 feet away from each other – the point at which the 'employee' should recognize the 'guest'. As soon as the first person stops, the second person should stop moving too.**
- **While the 'employee' and guest' are stopped the other group members measure the exact distance between them.**
- **Once the measurement has been taken, the pair should carry on until they feel it would be natural to say 'hello' or 'good afternoon'. Stop again.**
- **The group repeats the measurements to assess the accuracy of the pair's estimate of 5 feet.**
- **Try this experiment with different combinations of people from the group to see who achieves the best estimates for distance.**

Can you think of any cultures where the 5/10 rule might not work? If you can, explain the different practices in some other cultures for speaking to strangers, making eye contact and observing personal space.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion D: Applying mathematics in real-life contexts.

How much can we rely on models?

Model

A depiction of a real-life event using expressions, equations or graphs.

ACTIVITY: How many miles are in a kilometre?

ATL

- Critical thinking skills: Practise observing carefully in order to recognize problems

The graph shows the relationship between kilometres and miles: the number of miles versus the number of kilometres for the same distance.

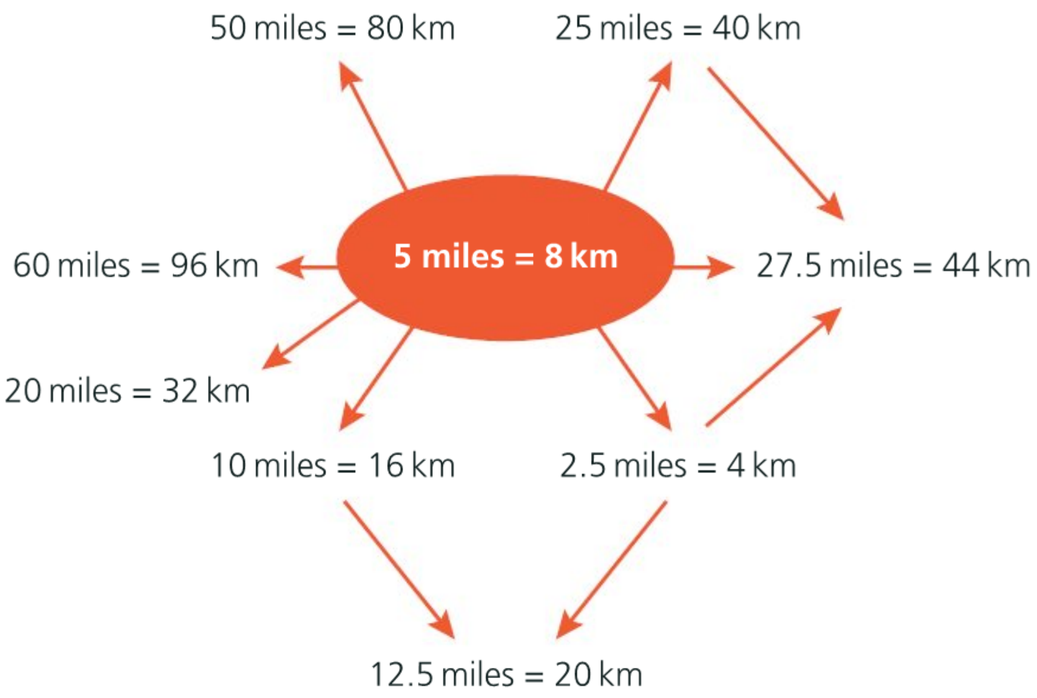
We can tell from the graph that 40 kilometres matches, or is equal to, 25 miles. We can make an equation from this information:

25 miles = 40 kilometres

Which means that:

5 miles = 8 kilometres

This means we can work out other equivalent relationships, such as:



Using the graph and the image above, find the missing values to complete the table. You will have to convert from one unit to another.

Distance (miles)	Distance (kilometres)
10	
100	
	80
	32
1000	
	400
	4000
2.5	
	16000



Here is a poster advertising an exhibition of Robin du Puy's work. The display shows the photographs taken during the artist's motorbike tour across the US.

There's a (mathematical) problem with this picture. Can you find it?



Write a letter to the museum to inform them of their mistake. Explain the mistake very carefully to them and explain what they should have said. Use mathematical communication and reasoning in your answer.

What other mathematical symbol can you see on the poster? What does it mean? Why might this photography museum have chosen it for their logo?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

How can we group numbers together based on their quantities (in dots)?

TYPES OF NUMBERS

Quantity

An amount or number.

For the next few activities we will convert numbers into dots – a quantity of dots matches a particular number. For example:



We do this so we can manipulate the dots. When we move them around we can see their **properties** and patterns.

For example, a square number is one which is formed when a number is multiplied by itself.

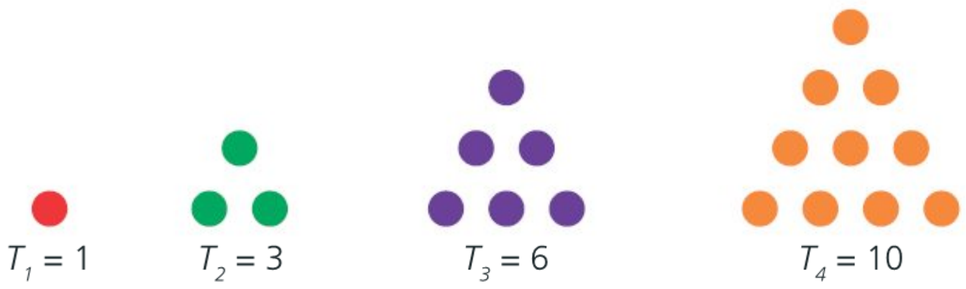
Name the first 10 square numbers. Try to do them from memory, but use your knowledge of multiplication if you can't remember them quickly.

Could you remember them all? We can represent these square numbers as actual visual squares, where each dot counts as 1.



But what if we wanted to make **triangles** instead of squares? Which numbers would have the right quantity of dots to make a complete triangle?

Here you can see the first four triangular numbers:



These triangular numbers go on forever, increasing by adding another row. Can you find the first ten triangular numbers?

Write them as a list, as well as visually.

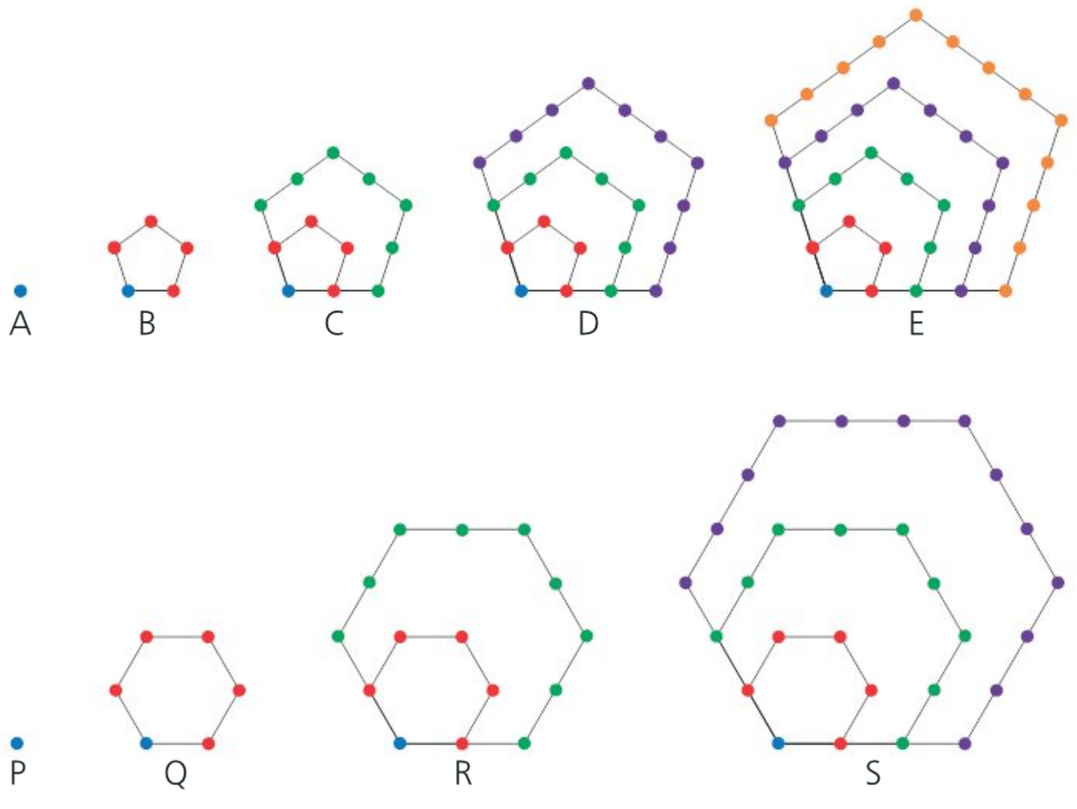
WHAT OTHER 'SHAPE NUMBERS' CAN WE FIND?

We are now familiar with triangular and square numbers, so let's look at the next possible 'shape number' – pentagonal numbers.

Based on this image showing five shapes, A to E, completed with dots, what are the first five pentagonal numbers?

Based on the shapes P to S, what are the first four hexagonal numbers? How could you calculate these?

Can you think of any other type of number that could be found or named in this way?



PARTITION NUMBERS

A partition is something which divides things, for example a partition wall can be constructed to divide up a room.

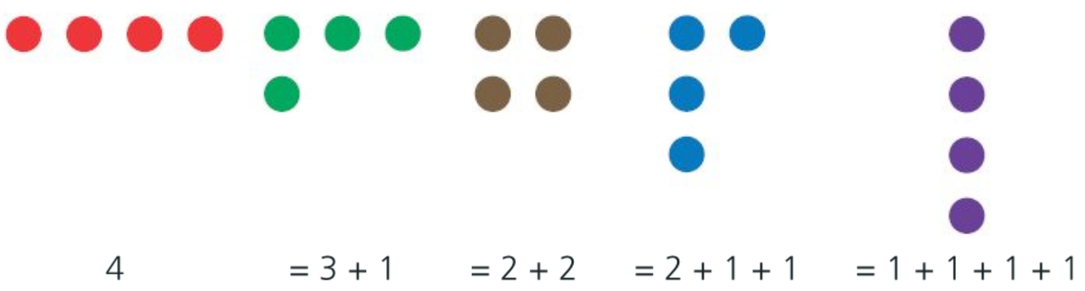
A **partition number** is a way of writing a number as the sum of positive integers (whole numbers). The partition number of 4 is 5, or

$p(4) = 5$

because 4 can written as the following 5 sums of numbers:

1. 4
2. 3 + 1
3. 2 + 2
4. 2 + 1 + 1
5. 1 + 1 + 1 + 1

We can also show this visually:



PRACTICE EXERCISE

By drawing dots, or using moving counters or coins, find out what the partition number is for 3.

There are 7 partitions for the number 5. We can write this as $p(5) = 7$. Can you draw them all?

Find $p(6)$.

As you can see, the partition numbers get very large very quickly. To learn more about partition numbers watch **www.youtube.com/watch?v=NjCIq58rZ8I** or search online for **numberphile, partitions**.

An interesting movie to watch is 'The Man Who Knew Infinity', in which partition numbers feature as a large part of the plot!

EXTENSION

What is a kissing number?



How can you represent your learning creatively?

Representation

The manner in which something is presented.

ACTIVITY: Creating a clock

ATL

- Creative-thinking skills: Create original works and ideas; Use existing works and ideas in new ways

Suppose your school wishes to replace a clock with an **innovative** version that shows off the knowledge of each year group. Instead of numbers, the clock will have short mathematical questions that give answers of 12, 1, 2, 3, etc. for the hours.

You have been chosen to create the questions to represent MYP1 (Grade 6). Your headteacher (principal) wants a few questions from each unit you have studied. They are particularly interested in having some algebra questions, because they think it looks impressive.

Make sure you double-check that each question gives a whole, positive and correct answer!

Create your clock on a circular piece of paper. Use your compass or an alternative method to draw a circle and cut it out.



A mathematics clock at the International School of Hamburg

Pattern

A set of numbers or objects that follow a specific order or rule.

HOW DO MAGIC SQUARES WORK?

What do you notice about the arrangement of these numbers into a 3 × 3 box? Can you work out a pattern?

4	9	2
3	5	7
8	1	6

Can you find another arrangement of numbers which will follow a similar pattern, also fitting into a 3 × 3 box?

These types of squares are known as **magic squares**. Magic squares were known to Chinese mathematicians as early as 650 BCE and to Islamic mathematicians possibly as early as the seventh century CE.

The story of Lo the Lo Shu (洛書) or 'scroll of the river Lo' dates back to 650 CE. According to the legend, a huge flood hit ancient China. King Yu tried to channel the water out to sea, when a turtle emerged from the flood. The turtle's shell had a curious pattern. It was a 3 × 3 grid with a pattern of circular dots representing numbers. The numbers in each row, column and diagonal line always added up to 15. The legend says that the people were able to use the pattern to control the river and protect themselves from floods. Every normal magic square of order three is obtained from the Lo Shu by rotation or reflection.

Find numbers which will satisfy (meet the conditions) of a magic square of the order of 4. You will need to draw a 4 × 4 grid.

EXTENSION

Using the numbers in the 4 × 4 grid as answers, create an algebraic expression for each box in the grid. For example, instead of using the 15 in the grid, you could write this expression: $2x - 1 = 29$

How can we name and identify shapes all around us?

Space

The frame of geometrical dimensions describing an entity.

ACTIVITY: What shape is a football?

ATL

- Critical thinking skills: Analyze complex concepts and projects into their constituent parts and synthesize them to create new understanding

Look at these pictures of a football and a truncated icosahedron. Why do some people say that a football is a 'spherical analogue of a truncated icosahedron'?



Research each individual term to find out what it means:

- **spherical**
- **analogue**
- **truncation/truncated**
- **icosahedron.**

In your own words, explain how these words, put together, accurately describe the shape of the ball. Do you think mathematics really is all around us?

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

ACTIVITY: Swimming in shapes

ATL

- Transfer skills: Apply skills and knowledge in unfamiliar situations

Look at this list of cool and unusual words:

- **dodecahedron**
- **frustrum**
- **myriagon**
- **megagon**
- **octakischiliagon**
- **octothorpe**
- **ellipse**
- **torus!**

Find out what each of these shapes looks like and **draw** an example. Find more weird and funny names for shapes. Did you already know some of these shapes or are they new to you? Can you find examples of these shapes in the real world?

Can you find these shapes in the real world to make a photo collage?

Now that you know the names of lots of new shapes, go on a scavenger hunt (a search to find objects in places all around you) to find them. Look for them in surprising and innovative places. Take as many photographs of these shapes as you can.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

Take action

- Display your photos from the scavenger hunt in a creative and imaginative way.



In this photo it appears that the greenhouse in the Botanical Gardens in Hamburg is a perfect pyramid.



However, up closer and from a different angle, it appears that the pyramid has been cut in two. Can you describe these shapes?

ACTIVITY: Bermuda's Good Friday kites

ATL

- Communication skills: Use intercultural understanding to interpret communication

Legend has it that a Bermudian teacher, in need of a simple yet effective way to demonstrate the Ascension of Christ into heaven on the Christian festival of Easter, used a kite decorated with an image of Jesus to imprint the concept on the minds of his students. As a result, on Good Friday in Bermuda, colourful, multi-sided kites made of sticks and tissue paper dot the skies all over the island.

This image shows an example of typical Bermuda kites from Somersfield Academy. **Describe** the shapes of the kites, using correct mathematical terminology. Are they all the same?



Using the typical Bermudian kite structure shown in the photo, design a Good Friday kite which includes examples of the new polygons you have learned and researched about.

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

How can we place numbers and things into a Venn diagram?

HIGHEST (GREATEST) COMMON FACTOR

We often look for the highest (or greatest) common factor (HCF or GCF) of two numbers.

Take the numbers 36 and 50. The factors can be written as:

36: (1), (2), 3, 4, 6, 9, 12, 18, 36

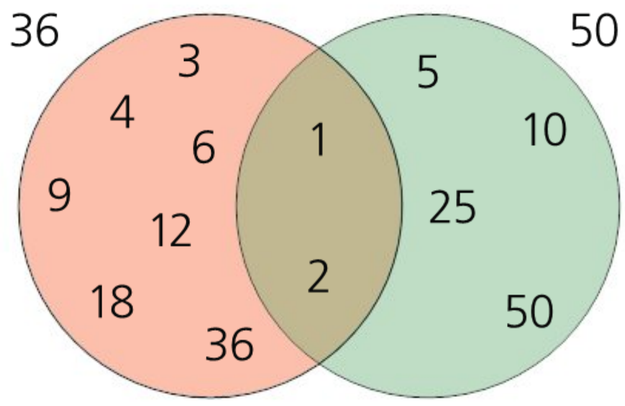
50: (1), (2), 5, 10, 25, 50

The common factors are circled. Therefore, the HCF of 36 and 50 is the factor 2. We can also show this information on a Venn diagram. A Venn diagram is a way to represent different sets or elements. If the two sets have something in common, you place that on the **intersection** or the overlap between the two circles.

If we place all the factors of 36 in the first circle and all the factors of 50 in the second circle, we must put the two common factors in the shared space in the middle. The HCF is 2.

System

A group of interrelated elements.



ACTIVITY: Going up to find the lowest

ATL

- Creative-thinking skills: Practise visible thinking strategies and techniques

Using a 100 square, place a red circle in each multiple of 7 up to 100. Now draw a blue star on each multiple of 9. What do you notice? What do we call this?

Using the listing technique, list out all the multiples for both 7 and 9 up to 100. Circle the **first (or lowest) common multiple** (FCM or LCM). Can you represent this information on a Venn diagram?

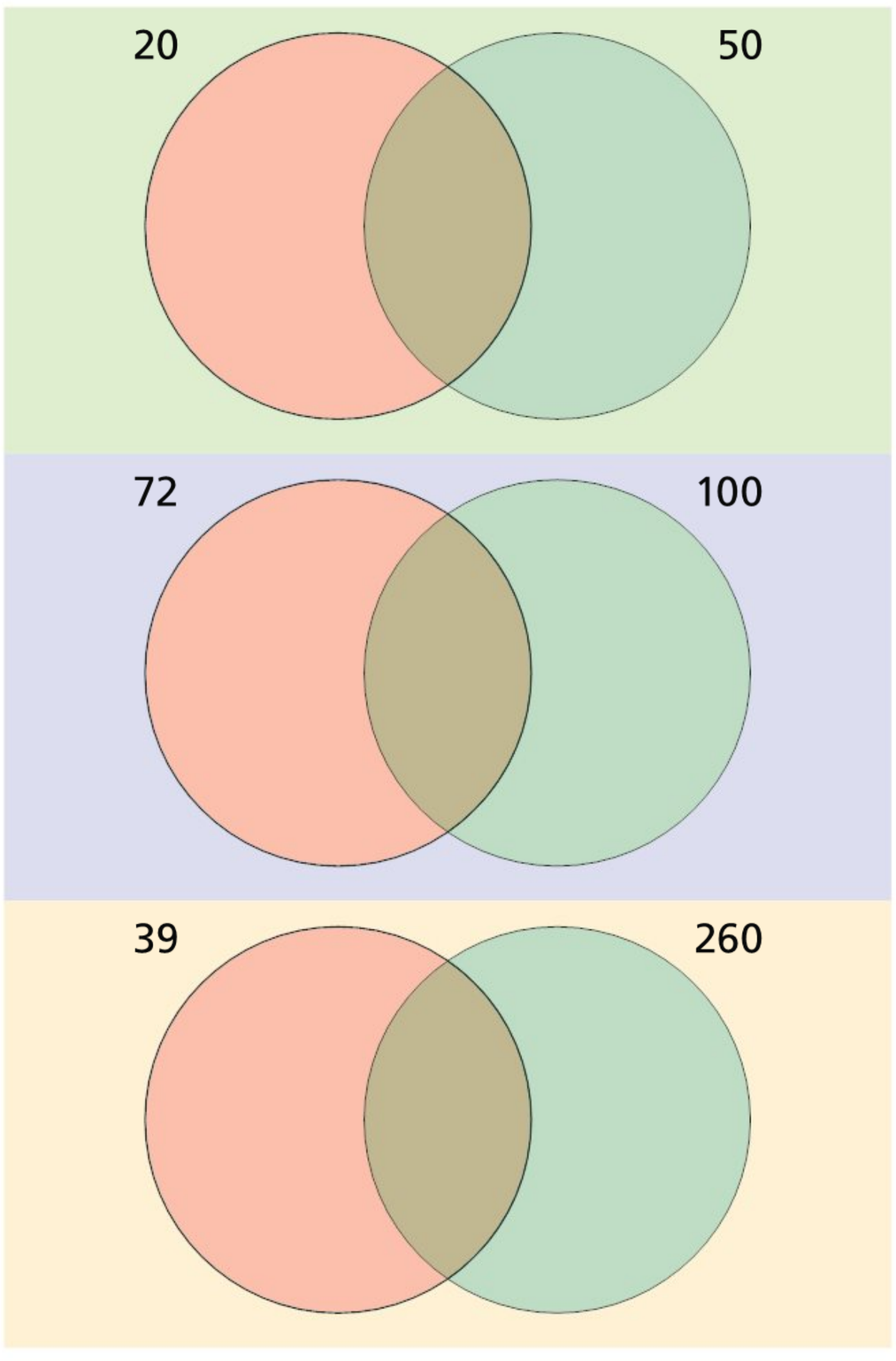
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion C: Communicating.

PRACTICE EXERCISE

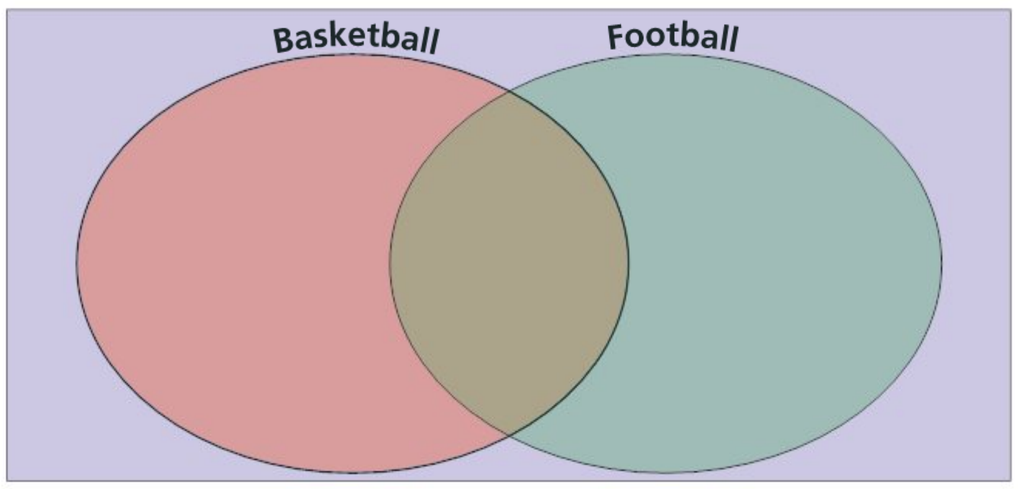
Complete these Venn diagrams with the factors for each number. Place the common factors in the intersection. Circle the HCF if there is more than one common factor.



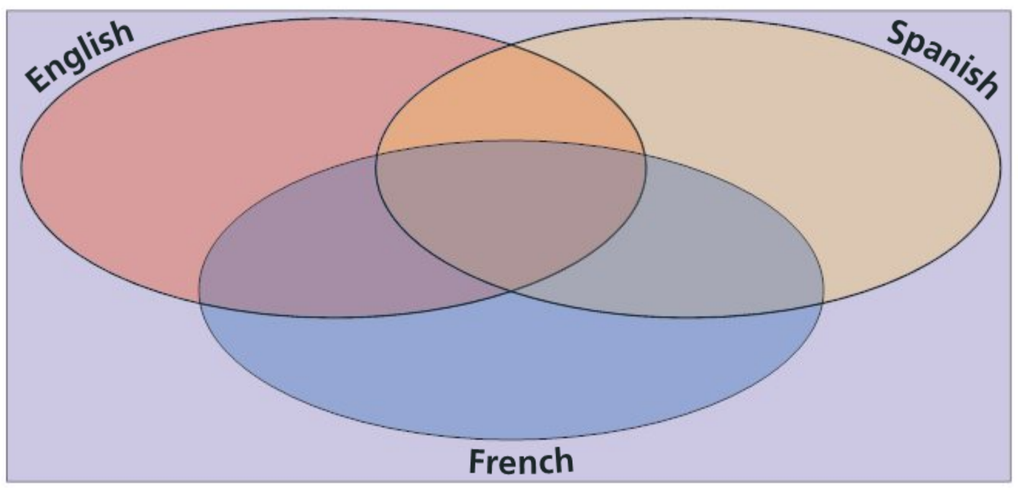
ACTIVITY: Venn diagrams

Complete these Venn diagrams.

1 Copy and complete the following diagram to show what these two sports have in common and how they are different to one another.

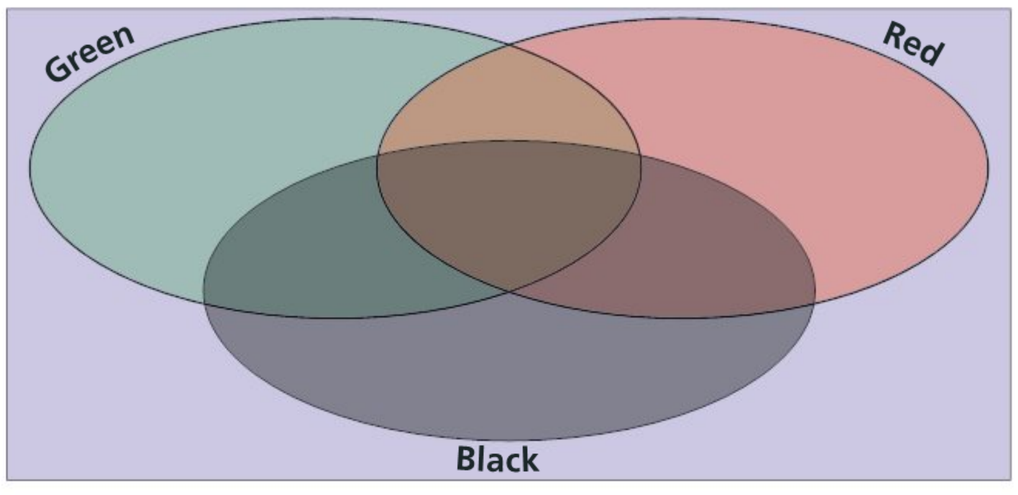


2 Complete the diagram by identifying words that French, English and Spanish have in common, as well as some that are different.



3 Place each country in the correct part of the Venn diagram, based on the colours in their national flag.

- Jamaica
- China
- USA
- Ghana
- Lebanon
- Senegal
- Jordan
- Germany



Assessment opportunities

- In this activity you have practised skills that are assessed using Criterion B: Investigating patterns.

MEET THE MATHEMATICIANS: KATHERINE JOHNSON (1918–), DOROTHY VAUGHAN (1910–2008) AND MARY JACKSON (1921–2005)

Learner profile: Risk-takers

It's time to finish the school year on a high … let's go to outer space!

Why not have some fun on your last day by choosing an amusing, mathematically themed movie to finish off your first year of MYP?

Released in January 2017, over the weekend of the Martin Luther King celebrations in the US, 'Hidden Figures' was a box-office hit and warmly received. Hidden Figures tells the story of three mathematicians, Katherine Johnson, Dorothy Vaughan and Mary Jackson, who were part of NASA's team of human 'computers'. These women were not only crucial to the success of the first orbital mission (in which the first human orbited the Earth) but they were also amazing risk-takers. They were talented, but often discriminated against because of their gender and/or colour. Individually and together they forged a difficult pathway and advanced the cause of human and scientific endeavour.

Would you like to know more about these incredible mathematicians? Why not try this for some holiday reading: *Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race*, by Margot Lee Shatterly?

Online more your thing? Check out **www.futurekatherinejohnsons.com/** and **www.nasa.gov/modernfigures** for some inspirational reading.









Reflection

Use this table to reflect on your own learning in this chapter.					
Questions we asked	Answers we found	Any further questions now?			
Factual: How does the order change the answer? How do magic squares work? How can we name and identify shapes all around us? How can we place numbers and things into a Venn diagram?					
Conceptual: How and when are mathematical innovations made? How good is your estimation? Why simplify? How can we group numbers together based on their quantities (in dots)? How can you represent your learning in a creative and innovative way?					
Debatable: Is there ever a 'best' method? Can mathematics make the perfect picture? How much can we rely on models?					
Approaches to Learning you used in this chapter:	Description – what new skills did you learn?	How well did you master the skills?			
		Novice	Learner	Practitioner	Expert
Communication skills					
Creative-thinking skills					
Collaboration skills					
Transfer skills					
Critical-thinking skills					
Learner profile attribute(s)	How did you demonstrate your skills as a risk-taker in this chapter?				
Risk-taker					

Glossary

absolute value The value of a number, ignoring or disregarding the positive or negative sign

acute An angle that measures between 0° and 90°

angle A figure formed by two lines or line segments, measured in degrees

area The amount of space inside a two-dimensional shape

census A system of recording and analysing data from an entire population

circumference The distance or length around a circle

classify To group, or put objects and numbers into different classes

coefficient A number used to multiply a variable, indicated in front of the variable

commutative An operation where the order is not important

composite figure A shape or figure made up by smaller, simpler shapes or figures

conjecture A statement that has not been proven

conjoining Incorrectly joining terms in algebra

consecutive Following each other, one after another

constant A number that does not change; not a variable

convention Agreed and commonly accepted notation or communication for mathematicians

cross section The area revealed by cutting an object through the centre

decimal A number or numerical quantity that is not an integer or whole number; expressed as a number of base 10; using a decimal point to indicate place value

deduce Draw a conclusion or derive statements

degrees A unit of measurement for angles; 1/360th of a circle or of a full revolution

diameter The length of a straight line segment that extends through the centre and ends at the circumference of a circle

divisor A number that is being divided into another number

equation A statement that makes two mathematical expressions equal

equilateral A triangle with three equal sides

equilibrium Balance or state of equality

estimate To obtain an approximate value for an unknown quantity

exponent A power to which a number has been raised; has been multiplied by itself

expression An expression in algebra is a grouping or phrase of algebraic terms, which might include one or several variables and constants

factorial The product of an integer and all the integers below it, as far as 1

fraction A number or numerical quantity which is not an integer or whole number, expressed as a quotient

greatest common factor The highest number that can be divided equally into two, or more, whole numbers

highest common factor The highest number that can be divided equally into two, or more, whole numbers

histogram A diagram that shows frequencies, whereby each interval is of equal width and there are no gaps between the bars

implied Without being stated, assumed or expected

improper A fraction where the numerator is larger than the denominator

index A power to which a number has been raised, has been multiplied by itself (plural, indices)

indices Plural of index

integers All whole numbers, both positive and negative

interior angle Angle inside a polygon

internal angle Angle inside a polygon

irrational number A number that cannot be expressed as a ratio or fraction; a non-terminating decimal value

irregular A shape with non-identical sides or angles

isosceles A triangle with two equal sides

least common multiple The lowest number that appears in lists of multiples for two or more numbers

line segment Part of a line with two ending points

lowest common multiple The lowest number that appears in lists of multiples for two or more numbers

mass A measure of the amount of matter in a body

maximum The value of the highest number in a set of data

minimum The value of the lowest number in a set of data

mixed number A number made up of a whole number and a fraction

natural number A positive integer; any whole number starting from 1

negative number A whole number below zero; a negative integer

nominal See *qualitative*

non-recurring numbers A decimal (or fraction) whose value never repeats and never terminates

obtuse An angle that measures between 90° and 180°

open-ended Investigations that can be taken in multiple directions, with a variety of possible correct answers

operation Basic skill or instruction in mathematics, telling how to handle two or more numbers

outlier A datum that lies outside the expected values

percentage A representation of a number as a proportion out of 100

perimeter The sum of all sides of a polygon

polygon A shape bounded by straight lines

power A number to which a base number has been raised; has been multiplied by itself

primary data Data that has been collected by you, or directly from the source

product The number that results from multiplication of two or more numbers or letters

property A characteristic belonging to an object or a pattern in a group of objects

proportion A part of something in relation to or compared to a whole

protractor A tool for measuring and constructing angles

qualitative Data that does not have a numeric value, which is descriptive in nature

range The value which is given by subtracting the lowest value from the highest value in a dataset

rational number Rational numbers include all integers and everything in between; any number that can be expressed as a fraction

regular A shape or polygon with identical sides and angles

right angle An angle of 90°

root The root of a number x is another number, which when multiplied by itself a given number of times, equals x

scale The ratio or legend for enlargement

scale factor The amount by which something has to be enlarged, reduced or scaled

scalene A triangle with no equal sides

simplify Make a problem or equation as easy as possible by applying mathematical principles such as cancelling or collecting terms

simultaneously At the same time

spread How close or how far all the data are from each other; a measure of how consistent the data are

substitution Replacing a value for a variable

term Can be a variable, a constant, or a combination of both; terms are separated from one another by a + or a – sign

unknown Something which we do not yet know or have not solved or substituted for

variable Something that can vary or change in value

vertex The point or edge of a polygon

vinculum The line dividing the numerator and the denominator

whole number A number with no fractions or decimals; positive integers including zero

yield Result in; will give an answer or result

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